Problem set, Micro 3. Week 20: Asset pricing – continued, and aggregate demand

1. From last week:
   (a) Varian exercise 20.1.

2. APT, CCAPM and CAPM
   (a) Make sure you understand the example on p. 381 on “Expected utility and the APT.”
   (b) Consider the one-factor APT model

   \[ \hat{R}_a = b_0 + \beta_a \hat{R}_m + \epsilon_a. \]

   The single index is the return on the market portfolio. Assume that a riskless rate of return exists and that asset specific risk \( \epsilon \) can be diversified away. Show that expected returns in the APT equilibrium is, in fact, consistent with the CAPM equation.

3. Aggregate demand (somewhat hard). Suppose there are \( n \) consumers and \( J \) goods. Consumers differ only by their income levels \( m_i \). Denote the indirect utility function of consumer \( i \) by \( v(p, m_i) \) and the corresponding demand function by \( x(p, m_i) \). Show that if for any distribution of income levels \( (m_1, \ldots, m_n) \) aggregate demand can be written as a function only of \( p \) and aggregate income \( M = \sum_{i=1}^{n} m_i \) and if every consumer’s preferences are homothetic, then all consumers have identical preferences. (If you need to refresh what homothetic means, consult Varian ch. 9.2).

4. (If time left) Consider the following figure (next page) of an economy with two consumers 1 and 2. State the weak axiom of revealed preference. Do individual demands satisfy the weak axiom of revealed preference? Does average demand satisfy WARP?
Figure 1: WARP and aggregation