Problem set, Micro 3. Week 19: Asset pricing – continued

1. From last week:
   (a) Varian exercise 11.6
   (b) Varian exercise 11.7

2. APT and no-arbitrage: Consider a two-index model of asset prices

   \[ \tilde{R}_a = b_{0a} + b_{1a}\tilde{f}_1 + b_{2a}\tilde{f}_2 \]

   where asset specific risk is assumed to be diversified away. An investor considers three portfolios, A, B and C with the following characteristics:

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected return</th>
<th>( b_{1a} )</th>
<th>( b_{2a} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15</td>
<td>1.0</td>
<td>0.6</td>
</tr>
<tr>
<td>B</td>
<td>14</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>0.3</td>
<td>0.2</td>
</tr>
</tbody>
</table>

   (a) From geometry we know that three points determine a plane, in the same way that two points determine a line. Derive the equation for the plane spanned by the three portfolios. (Hint: Consider the relationship \( R_a = \lambda_0 + \lambda_1 b_{1a} + \lambda_2 b_{2a} \) for each asset and solve three equations in three unknowns).

   (b) Consider a portfolio \( D \) constructed by placing 1/3 in each of the three portfolios. Does this lie in the plane spanned by A, B and C. Calculate the \( b_{ia} \)'s for \( D \) and the expected return.

   (c) Consider a portfolio \( E \) with \( b_{E1} = b_{E2} = 0.6 \) and expected return = 15 %. Does this portfolio lie in the plane. If yes, show how! If no, interpret the result in terms of the no-arbitrage condition.

   (d) Show that by selling \( D \) short and investing in \( E \), an investor can generate a profit without risk (Hint: Consider the initial and end-of-period cash flows and well as the risks of \( D \) and \( E \) respectively).


4. (If time) Varian exercise 20.2.