
Note: Varian’s chapter 11 deals with uncertainty and expected utility, which I take as being known – though perhaps not at this level – from Micro 2. For those who feel a need to reaquaint themselves with choice under uncertainty, please do so from your Micro 2 lectures notes. I furthermore recommend pages 184-190 (from “Example” to section 11.8 (included)). I include a few useful exercises from Varian’s chapter 11 below.

1. Varian exercise 11.6
2. Varian exercise 11.7
3. Deriving the efficient frontier: In Varian’s figure 20.1 (and, to some extent, at the lectures on Tuesday), the shape of the efficient frontier is postulated, rather than derived. The purpose of this problem is to look in more detail at this frontier, as its shape helps us understand why diversification is good.

Consider a setting with two assets, $A$ and $B$. The portfolio $x$ has expected returns

$$\bar{R}_P = x\bar{R}_A + (1-x)\bar{R}_B.$$ 

(a) Write up the standard deviation of the return of the portfolio. Call this $\sigma_P$.

Recall the fact that $\sigma_{AB}$, the covariance between returns on $A$ and $B$, is given by

$$\sigma_{AB} = \sigma_A \sigma_B \rho_{AB}$$

where $\rho_{AB}$ is the correlation coefficient between assets $A$ and $B$. Using this, we now want to derive the shape of the so-called portfolio possibilities curve (the curve in figure 20.1) as a function of $\rho_{AB}$.

(b) Examine the relationship between $\bar{R}_P$ and $\sigma_P$ for $\rho_{AB} = +1, -1$ and 0, respectively.\(^1\) Aiding you in your derivations, you might want to consider an example that you can graph. Consider for example two assets with $\bar{R}_A = 14\%$, $\sigma_A = 6\%$, $\bar{R}_B = 8\%$ and $\sigma_B = 3\%$. For each of the cases for $\rho_{AB}$, vary $x$ from 0 to 1 and graph the results in the $(\sigma_P, R_P)$-space.

(c) Identify the portfolio $x^{mv}$ with minimum variance for arbitrary $\rho_{AB}$. How does this vary with $\rho_{AB}$?

(d) For each of the three cases of $\rho_{AB}$ from (b), identify graphically the efficient frontier of the set of risky assets, recalling the shape of indifference curves in the $(\sigma_P, R_P)$-space.

4. (If time permits) Varian exercise 11.12

\(^1\)For $\rho_{AB} = 0$ you might be better off finding $\sigma_P$ as a function of $R_P$ rather than the other way around.