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**Too Much of a Good Thing? The Quantitative  
Economics of R&D-driven Growth Revisited**

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# Too Much of a Good Thing?

## The Quantitative Economics of R&D-driven Growth Revisited

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**Abstract.** This paper augments an R&D-based growth model of the third generation with human capital accumulation and impure altruism, calibrates it with U.S. data, and investigates whether the market provides too little or too much R&D. For benchmark parameters the market share of employment in R&D is close to the socially optimal allocation. Sensitivity analysis shows that the order of magnitude of possible deviation between market allocation and optimal R&D is also smaller than suggested by previous studies. Furthermore, the model allows for two additional channels through which population growth may affect the resource allocation so that its overall economic impact is no longer predetermined as being positive. Numerical calibrations show that economic growth at the U.S. average rate during the last century can be consistent with a small and probably negative partial correlation between population growth and economic growth.

*Keywords:* Human Capital, Population Growth, Endogenous Economic Growth, R&D-Spillovers.

*JEL Classification:* J24, O31, O40.

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## 1. INTRODUCTION

Research as a market activity naturally implies that resources are suboptimally allocated. While knowledge spillovers and imperfect appropriation of rents by innovators result in too little R&D effort from a social perspective, creative destruction and duplication externalities work in the opposite direction. Negative and positive externalities compensate each other only by chance so that the question occurs whether the market provides too much or too little R&D and how large the deviation from the socially optimal solution will be. Estimates can be given using numerical parameterizations of R&D-driven growth models with actual data. This way, calibrations of endogenous and semi-endogenous growth models have found rather large deviations of market solutions from the socially optimal one. The current paper challenges these findings by investigating an R&D-driven growth model of the third generation augmented by human capital accumulation.

According to Jones (1999) economic growth theory with endogenous technological change can be classified into three types or generations of models. Models of the first type are developed by Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). Long-run growth is generated by knowledge spillovers of degree one implying a positive correlation between per capita growth and population size. Criticism of this unobservable scale effect has led to the development of second-generation growth models by Jones (1995), Kortum (1997), and Segerstrom (1998). Two common features of these models are knowledge spillovers of less than degree one and positive population growth. Spillovers of less than unity eliminate the scale effect of population size but they also require perpetually increasing research effort for a steadily positive rate of innovation. Given that a constant fraction of population is engaged in research this requirement is satisfied by population growth.

Models of the first generation imply that the market usually generates suboptimal growth through the above mentioned channels. In second-generation models the same growth rate is realized by market and social planner. Economic growth is semi-endogenous because it is endogenously explained but driven by parameters which are usually considered as exogenous. Nevertheless, the market provides suboptimal R&D because externalities and imperfections have *level effects* on resource allocation between R&D and other activities.

Stokey (1995) calibrates a first-generation growth model with quality research and Jones and Williams (2000) investigate a numerically specified semi-endogenous growth model with

variety research. Both articles find that the optimal share of resources devoted to R&D deviates in large orders of magnitude from the market solution. For example, letting the duplication externality vary between zero and one Stokey finds that the share of resources optimally allocated to R&D lies between 70 percent below and 500 percent above the laissez-faire solution. Jones and Williams find optimal shares between 30 percent below and 400 percent above market shares.

While models of the first and second generation investigate either variety research or quality research, a third generation of endogenous growth models considers both activities simultaneously. Common features of earlier third-generation models, developed by Young (1998), Dinopolous and Thompson (1998), and Peretto (1998), are knowledge spillovers of degree one in improving quality of products but no spillovers in the variety dimension. A growing population triggers entry of new firms in variety research as in models of the second generation. If population stays constant, however, research specializes in improving existing products thereby realizing increasing returns and generating sustainable growth.

At first sight these models bring back the endogenous growth result and eliminate the need of population growth for economic growth. Yet, investigating a general two-R&D-sector model with externalities from both research activities Li (2000) demonstrates that these results hinge on two knife-edge conditions for knowledge spillovers. The general case of inter-sectoral spillovers renders growth to be semi-endogenous again and to rely necessarily on population growth.

While the knife-edge argument has found much attention in the subsequent literature another feature of Li's model remained relatively unexplored and is the focus of the present paper. The existence of two interdependent R&D sectors may lead to a reassessment of the deviation of market research effort from the socially optimal level. In a two-sector R&D model changing an assumption about an underlying externality may have mainly substitution effects and less impact on overall research effort. Consider, for example, an increase of knowledge spillovers in quality research. Taken the external effect into account a social planner would then allocate more researchers to this activity than the market. However, he may simultaneously allocate less researchers to variety research, so that the impact on overall R&D effort can less easily be assessed than suggested by previous studies. The present paper therefore calibrates the general two-R&D-sector model with U.S. data and re-investigates whether the market provides too much or too little R&D and how large a deviation from the socially optimal one can be expected.

The second main topic of the paper is a re-investigation of the role of population growth in economic growth. For that purpose it augments the R&D-driven growth model with human capital accumulation and a measure of altruism in preferences. Because it is now effective labor, i.e. the number of researchers multiplied by their skill level, that is engaged in research, R&D growth and thereby economic growth do no longer necessarily depend on population growth. For constant population growing research output can be solely driven by human capital accumulation and economic growth becomes fully endogenous again. Human capital growth reduces the importance of R&D because growth of income per capita is no longer driven by TFP growth only but also by labor quality growth. This reduces the role of R&D-externalities and provides a further reason for why the deviation of market R&D from the social optimum could be smaller than suggested by earlier models.<sup>1</sup>

Furthermore, while the economic impact of population growth through effective labor remains positive (as in existing R&D models), two channels appear through which a growing population affects growth negatively. A growing population dilutes the skill level per capita, raises the opportunity costs of education, and causes slower human capital growth. A second channel originates from abandoning the assumption of Benthamite utility according to which households are purely altruistic towards future generations. With less than pure altruism population growth operates through its dilution of financial wealth per capita similar to depreciation. It leads to a discount of perceived interest rates and has through this channel a negative impact on economic growth. Numerical calculations using the model calibrated with U.S. data investigate the quantitative outcome of the interplay of these three channels and challenge the previously found result of a large positive correlation between population growth and economic growth.

The next section briefly resumes essential components and analytical results for the general two-R&D-sector model with human capital accumulation. A detailed derivation and discussion of the theoretical model is available in Strulik (2005). Section 3 presents the calibration and benchmark results. Section 4 performs the sensitivity analysis and a final section concludes.

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<sup>1</sup>Employing different arguments two recent articles also find that the actual R&D intensity may be closer to the social optimum than originally suggested by Stokey (1995) and Jones and Williams (2000). Comin (2004) arrives at this conclusion by introducing exogenous technological progress in the Jones-Williams setup. Steger (2005) generalizes a second-generation model to the extent that capital goods are essential in all sectors of the economy. This paper emphasizes that a small deviation from optimal R&D is compatible with large welfare losses in the laissez-faire solution.

## 2. THE MODEL

Human capital, which replaces raw labor in an R&D-driven growth model, can be allocated to employment and education. Human capital employed in production of goods ( $H_x$ ) is called workers and employment in quality and variety research ( $H_Q$  and  $H_n$ ) is called researchers. Output of final goods  $Y$  sells on a competitive market at a price of one. It is produced by differentiated intermediate goods using a constant returns to scale technology with elasticity of substitution  $\sigma > 1$ . After  $\kappa$  innovations in quality research a good  $j = 1, \dots, n$  is available at qualities  $q_{k_j}$ ,  $k_j = 0, 1, \dots, \kappa_j$  and used in quantities  $x_{k_j}$ . Hence, aggregate output is given by

$$Y = \left[ \int_0^n \left( \sum_{k_j=0}^{\kappa_j} q_{k_j} x_{k_j} \right)^{1-1/\sigma} dj \right]^{1/(1-1/\sigma)}. \quad (1)$$

Each intermediate good  $j$  is produced by a monopolistically competitive firm using a linear production function  $x_{k_j} = H_{x_j}$  and sold at price  $p_{k_j} = w\sigma/(\sigma - 1)$ . At time of invention  $\tau$  a good  $j$  has initial quality  $Q_\tau^{1/(\sigma-1)}$ , where  $Q := (1/n) \int_0^n q_{\kappa_i}^{\sigma-1} di$  is an aggregate quality index. After  $k_j$  quality innovations it has quality  $q_{k_j} = \gamma^{k_j} Q_\tau^{1/(\sigma-1)}$ . Assuming  $\gamma > \sigma/(\sigma - 1)$  implies that only goods of leading edge quality are supplied.

An improvement from quality level  $\kappa$  to  $\kappa + 1$  occurs with probability

$$\mu_{\kappa_j} = \frac{A_Q}{q_{\kappa_j}^{\sigma-1}} H_{Q_j}, \quad A_Q := An^{\alpha_1} Q^{\alpha_2} H_Q^{-\chi}, \quad (2)$$

where  $H_{Q_j}$  are researchers employed in quality research for good  $j$ . Firms take general productivity as given which is determined by the following externalities. The term  $Q^{\alpha_2}$  is the externality usually taken into account in third-generation growth models. It describes knowledge spillovers from quality research assuming that the aggregate quality index  $Q$  approximates economy-wide available knowledge about quality improvements. Here we follow Li (2000) and additionally allow for knowledge spillovers from variety research to quality research determined by the term  $n^{\alpha_1}$ . The term  $H_Q^{-\chi}$ ,  $0 \leq \chi < 1$ , captures the possibility of duplication externalities (stepping on toes effects). Aggregating over  $j$  and using the law of large numbers shows that the quality index grows at rate

$$\frac{\dot{Q}}{Q} = (\gamma^{\sigma-1} - 1) \frac{A_Q}{Q} \frac{H_Q}{n}. \quad (3)$$

New varieties are produced by researchers  $H_n$  according to

$$\dot{n} = A_n H_n \quad , \quad A_n = \frac{A n^{\beta_1} Q^{\beta_2} H_n^{-\chi}}{Q} \quad , \quad (4)$$

where  $\beta_1$  and  $\beta_2$  specify the degree of intra- and intersectoral knowledge spillovers in variety research and  $\chi$  measures the duplication externality. By placing the quality index in the denominator it is assumed that creation of a new product (with initial quality  $Q$ ) becomes increasingly difficult for rising aggregate quality.

Determining market equilibrium and aggregating over goods provides final output as

$$Y = (nQ)^{1/(\sigma-1)} H_x \quad . \quad (5)$$

The term  $(nQ)$  shows increasing returns to scale in aggregate production resulting from variety and quality expansion.

Households maximize intertemporal utility from consumption per capita  $c \equiv Y/L$ :

$$\int_0^\infty \left[ c^{1-\theta} / (1-\theta) \right] e^{-(\rho-m\lambda)t} dt \quad \theta > 1 \quad , \quad m \in [0, 1] \quad , \quad (6)$$

where  $L$  is population size. Population grows at a given constant rate,  $\lambda$ , which may be positive, negative, or zero. The time preference rate is denoted by  $\rho > 0$  and  $1/\theta$  is the intertemporal elasticity of substitution. A particularity of (6) is the parameter  $m$  that controls for the degree of altruism towards future generations. The literature usually considers only one of the border cases. For  $m = 0$  households maximize utility of consumption per capita (Millian type utility) and for  $m = 1$  they take equally into account consumption of all members of their dynasty (Benthamite type utility). Here I employ the idea of Nerlove et al. (1982) and allow for a continuum of intermediate degrees of altruism.

Households face the usual income budget constraint and a Lucas (1988)-type production function for education.

$$\dot{h} = \xi(h - h_e) - (\delta + \lambda)h \quad . \quad (7)$$

The parameter  $\xi$  measures productivity in education,  $h \equiv H/L$  is human capital per capita,  $h_e$  is employed human capital (in production and research activities), and  $\delta$  is the rate of depreciation of knowledge. Note that population growth operates similar to depreciation of

knowledge: without further effort in education a growing population dilutes a dynasty's human capital per capita.

Solving the households allocation problem provides the Ramsey rule

$$\theta g_c = r - \rho - (1 - m)\lambda = \frac{1}{\sigma - 1} [g_n + g_Q] + \xi - \delta - \rho - (1 - m)\lambda , \quad (8)$$

where right hand side equality originates from insertion of equilibrium interest rate and growth rate of wages. In an equilibrium of supply and demand  $Y = C = cL$ , and (5) implies

$$g_c = \frac{1}{\sigma - 1} [g_n + g_Q] - \lambda + g_{H_x} . \quad (9)$$

A balanced growth path is defined by constant sectoral shares of employment and constant growth of variety and quality. Aggregate growth of human capital is obtained from (7) and equilibrium on factor markets,  $H_e = H_x + H_n + H_Q$ . Differentiating (4) and (3) with respect to time provides balanced growth rates for R&D.<sup>2</sup>

$$g_H = \xi \left( 1 - \frac{H_x}{H} - \frac{H_Q}{H} - \frac{H_n}{H} \right) - \delta , \quad (10a)$$

$$g_n = \frac{\alpha_2 - \beta_2}{D} (1 - \chi) g_H , \quad (10b)$$

$$g_Q = \frac{\beta_1 - \alpha_1}{D} (1 - \chi) g_H , \quad (10c)$$

$$D = (1 - \alpha_1)(1 - \beta_2) - (1 - \alpha_2)(1 - \beta_1) .$$

From (8) – (10) equilibrium growth is uniquely determined.

$$g_c = \underbrace{\frac{\xi - \delta - \rho}{\theta - 1} \left( 1 - \frac{1}{\theta + \phi} \right)}_{a > 0} + \underbrace{\frac{1}{\theta - 1} \left[ m - \frac{\theta - (1 - m)}{\theta + \phi} \right]}_b \lambda, \quad (11)$$

$$\phi \equiv \frac{(\theta - 1)(1 - \chi) (\beta_1 - \alpha_1 + \alpha_2 - \beta_2)}{\sigma - 1} \frac{1}{D} + \theta .$$

As in growth models of the third generation, the economic growth rate can be represented as an affine linear function of population growth, where positivity of  $a$  follows from  $\theta > 1$  and  $\phi > 0$ . With contrast to that literature, however, the partial correlation between both growth rates – given by the parameter  $b$  – is not necessarily positive. For pure Millian preferences ( $m = 0$ ) the correlation is negative while it is positive if preferences are purely Benthamite ( $m = 1$ ).

<sup>2</sup>For positive growth either spillovers within the same sector have to be larger than across sectors  $\alpha_2 > \beta_2$  and  $\beta_1 > \alpha_1$  or the opposite has to be true,  $\alpha_2 < \beta_2$  and  $\beta_1 < \alpha_1$ .

The intuition for the ambiguity is as follows. Population growth affects economic growth through three channels, a positive scale effect, a negative human capital dilution effect, and a positive but possibly small time preference effect. The scale effect operates through the growth rate (instead of the size) of population and affects R&D just as in models of the second and third generation. Higher population growth leads to higher growth of aggregate human capital ( $g_H = g_h + \lambda$ ), and therewith to higher growth of human capital devoted to R&D, higher growth of research output (visible in (10)), and higher economic growth (through  $g_n$  and  $g_Q$  in (9)). Note, however, that population growth – with contrast to the interpretation delivered in connection with earlier models – does not lead to a higher number of geniuses (Simon, 1981) or Isaac Newtons (Jones, 2003) in society. As long as growth of skills per person,  $g_h$ , remains constant population growth just increases the number of generally talented researchers.

Population growth, however, affects growth of skills negatively. Because newborns enter the world uneducated, population growth dilutes the stock of human capital per capita (visible in (7)). In order to equip a growing population with a certain skill level a larger share of resources has to be allocated to education than in an otherwise identical economy with a stable population. When population grows at a higher rate, households face higher opportunity costs of education and allocate less resources to education, which results in a lower growth rate of human capital per capita. For households with purely Millian preferences the negative human capital dilution effect overcompensates the scale effect in R&D so that the overall effect of growth on economic growth is negative.

For altruistic households a time-preference operates against the human capital dilution effect because a larger future size of the dynasty increases the weight assigned to consumption per capita of later generations. More patient households invest more and more investment in R&D and human capital leads to higher growth. This can best be seen by a raising  $m$  in (8). When  $m$  approaches one only the positive scale effect from R&D growth remains. The time preference effect compensates the dilution effect if preferences are purely Benthamite ( $g_c > 0$  for  $m = 1$  in (11)). For intermediate cases of altruism, however, we cannot determine theoretically which effect dominates and a numerical specification of the model is needed for an assessment of the correlation between population growth and economic growth.

A social planner also faces growth rates (8) – (10) and would therefore realize the same economic growth rate as the market. He, however, takes the four types of externalities into

account and allocates factors differently to sectors. In Strulik (2005) it is shown that net returns in a market equilibrium with R&D in both sectors requires

$$\xi - \delta = \left[ \frac{H_x}{H_Q} \frac{1}{\sigma - 1} - 1 \right] \frac{g_Q}{\gamma^{\sigma-1} - 1} - \alpha_1 g_n - \alpha_2 g_Q + \chi g_H \quad , \quad (12a)$$

$$\xi - \delta = \left[ \frac{1}{\sigma - 1} \frac{H_x}{H_n} - \frac{H_Q}{H_n} \right] g_n - \beta_1 g_n - (\beta_2 - 1) g_Q + \chi g_H \quad , \quad (12b)$$

where the net interest rate is given  $\xi - \delta \equiv r - \dot{w}/w$ . Solving for the social optimum shows that it implies a factor allocation where (12) is replaced by (13).

$$\xi - \delta = \left[ (\beta_2 - 1) \frac{H_n}{H_Q} + \frac{1}{\sigma - 1} \frac{H_x}{H_Q} (1 - \chi) \right] g_Q - (\alpha_1 - 1) g_n + \chi g_H \quad , \quad (13a)$$

$$\xi - \delta = \left[ (\alpha_1 - 1) \frac{H_Q}{H_n} + \frac{1}{\sigma - 1} \frac{H_x}{H_n} (1 - \chi) \right] g_n - (\beta_2 - 1) g_Q + \chi g_H \quad . \quad (13b)$$

Equations (12) and (13) coincide only accidentally. Usually the planner's allocation deviates yielding a higher income level at each point of time and higher welfare than provided by the market solution. Without numerical specification, however, we cannot decide whether market R&D effort is too high or too low and how large the difference may be.

### 3. CALIBRATION

The calibration of the model with U.S. data follows related previous studies of numerical R&D models, in particular Jones and Williams (2000). The long-run rate of per capita income is set to 1.75 percent p.a. and the time preference rate to 2.0 percent. Based on the estimate of Mincer (1974) I set  $\delta = 0.01$ . According to Jones and Williams growth of the labor force is set to 1.44 percent and the steady-state interest rate is set to 7.0 percent representing the average real return on the stock market for the last century.

In the benchmark case we consider a medium degree of altruism,  $m = 0.5$ . Inspection of (8) shows that an economic growth rate of 1.75 percent implies a value of  $\theta$  of 2.45. Alternatively, for Millian ( $m = 0$ ) or Benthamite ( $m = 1$ ) preferences the implied values of  $\theta$  are 2.0 and 2.9. Hence, the whole range of possible elasticities is in an order of magnitude of values usually used in other calibration studies of economic growth models.

Turning towards market power, let  $\eta := \sigma/(\sigma - 1)$  define the markup. Since Jones and Williams consider markup factors between 1 and 1.37 we set a benchmark value of 1.2 and provide a sensitivity analysis of results with respect to other values. The benchmark value for

$\eta$  implies  $\sigma = 6$  and a knowledge elasticity of output of .2, exactly as in the Jones-Williams study. Similarly, since Jones and Williams consider duplication externalities between zero and one, we set  $\chi = 0.5$  in the benchmark case and investigate the impact of parameter variation. The parameter  $\gamma$  is specified so that the inverse of the rate of creative destruction  $\tau := 1/\mu = (\gamma^{\sigma-1} - 1)/gQ$  is consistent with estimates of the expected lifetime of a design. We consider lifetimes between 5 and 50 years corresponding to the range of values considered by Stokey and Jones and Williams and set the benchmark lifetime to 10 years.

Finally, also following Jones and Williams, we require that growth of total factor productivity  $(g_n + g_Q)/(\sigma - 1)$  equals 1.25 percent. From (8) we see that this ties down the value for general productivity in education ( $\xi$ ) to 0.0675.

With respect to knowledge spillovers we begin by setting  $\alpha_2 = 1$ . This assumption of spillovers of degree one within quality research is made by endogenous growth models of both the first and the third generation. As a starting point we furthermore assume that production of new ideas in both R&D sectors is structurally similar and impose symmetry,  $\beta_1 = \alpha_2$ ,  $\beta_2 = \alpha_1$ . Finally, we solve equations (8) – (10) which provides the equilibrium growth rates together with a value for the sole parameter left yet unspecified, the degree of knowledge spillovers between sectors,  $\alpha_1$ . For benchmark parameters we obtain  $g_H = 0.019$ ,  $g_n = g_Q = 0.31$  and  $\alpha_1 = 0.69$ . The model calibrated using U.S. data therefore suggests that knowledge spillovers between sectors are rather large but also clearly smaller than one.

TABLE 1: BENCHMARK PARAMETERIZATION AND RESULTS

Parameters and Steady-State Variables Determined										
$\rho$	$\delta$	$\lambda$	$g_c$	$g_{TFP}$	$r^*$	$\eta$	$\tau$	$\chi$	$\alpha_2$	$m$
.02	.01	.0144	.0175	.0125	.07	1.2	10	.5	1.0	.5
Parameters and Steady-State Values Implied										
$\alpha_1$	$\xi$	$\sigma$	$g_n$	$g_Q$	$g_H$	$\theta$	$b$			
.69	.0675	6.0	.031	.031	.019	2.45	-.053			
Market Solution					Social Optimum					
$H_x$	$H_n$	$H_Q$	R&D-Share ( $s^{DC}$ )		$H_x$	$H_n$	$H_Q$	R&D-Share ( $s^{SP}$ )		
49.3	2.24	4.94	7.18		50.0	3.27	3.27	6.54		

Factor allocation in percent of human capital endowment (potential work force), e.g.  $H_x = 50$  means 50 percent of human capital is allocated in manufacturing.

Given these values, we calculate the factor allocation for the decentralized economy ( $DC$ ) from (10a) and (12) and the optimal allocation chosen by the social planner ( $SP$ ) from (10a) and

(13). Aggregating both allocations we obtain the total share of employment in R&D  $s^{DC} = 7.2$  percent and  $s^{SP} = 6.5$  percent indicating that the market provides somewhat too much R&D. Besides allocating slightly less resources to R&D the planner chooses a different allocation of researchers to quality and variety research. Internalizing spillovers he allocates equal shares of 3.3 percent of total employment to both research activities while the market allocates 4.9 percent to quality research and only 2.2 percent to variety research. The main cause of deviation is the probability of success (i.e. the rate of creative destruction) which is internalized by the planner but not by the market solution. Because creative destruction reduces the time span during which a newly invented variety creates value for the initial inventor, the market allocates too few researchers to variety research and too much to quality research. Compared with earlier studies the overall deviation between market solution and social optimum one can be assessed as small. For duplication externalities of .5 Jones and Williams (Table 2) obtain an R&D ratio ( $s^{SP}/s^{DC}$ ) of 1.67. The present calibration calculates this share as 0.91.

Given the fully numerically specified model we can read off the implied correlation between economic and population growth from equation (11). The value for  $b$  is  $-0.0527$  indicating a slightly negative correlation close to zero. Doubling population growth from 1.4 to 2.8 percent would lower economic growth from 1.75 to 1.70 percent. This result of an almost insignificant impact of population growth contrasts sharply with earlier findings. In Stokey’s endogenous growth model of the first generation population growth would cause hyper- exponential economic growth. In Jones and Williams’ semi-endogenous growth model the coefficient of correlation must be by design larger than unity.<sup>3</sup> Table 1 summarizes benchmark parameterization and results.

#### 4. RESULTS OF SENSITIVITY ANALYSIS

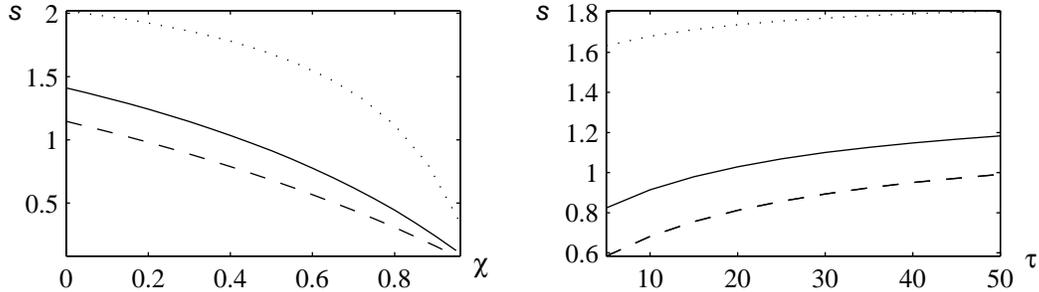
Given the large uncertainty about the true magnitude of the various external effects, point estimates of factor allocations are relatively useless. This is in particular true given the large sensitivity of results against parameter variation obtained by Stokey and Jones and Williams. We therefore follow these authors and investigate results in form of sensitivity analysis.

The left hand side of Figure 1 shows results with respect to alternative duplication externalities. The solid line assumes the benchmark interest rate. Because the “stepping on toes effect” is taken into account by the social planner, a higher duplication externality implies a lower

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<sup>3</sup>It can be obtained as 1.3 from their “typical calibration”, Table 2, p. 75.

FIGURE 1. R&D-Share: Social Planner/Market Solution ( $s$ )  
Against Duplication Externality ( $\chi$ ) and Lifetime of Design ( $\tau$ )



Solid Lines  $r^* = 0.07$ , Dashed Lines  $r^* = 0.1$ , and Dotted Lines  $r^* = 0.04$ . Other Parameters as in Table 1.

optimal share of R&D. For values of  $\chi$  above 0.45, the market allocates too much resources to R&D. In the corresponding scenario in Jones and Williams overinvestment occurs only at much higher externalities above 0.75.<sup>4</sup> Dotted and dashed lines show sensitivity with respect to the interest rate which is also the net return of R&D in the market solution. For low interest rates (returns on research) of 4 percent the market allocates too little resources to R&D except for high duplication elasticities above 0.8. Given high interest rates of 10 percent the market invests too much in R&D for duplication elasticities above 0.2.

Compared with results from Stokey (Figure 3) and Jones and Williams (Figure 1) the overall impression from the current analysis is that of less sensitivity of results against parameter variation. This finding has a straightforward intuition. With human capital accumulation the present model takes a further engine of growth into account. The new channel of factor quality growth mitigates the importance of R&D – the activity where the externalities are present – for economic growth. Hence, holding economic growth constant at benchmark values, varying the size of the externality has a smaller impact on factor allocation.

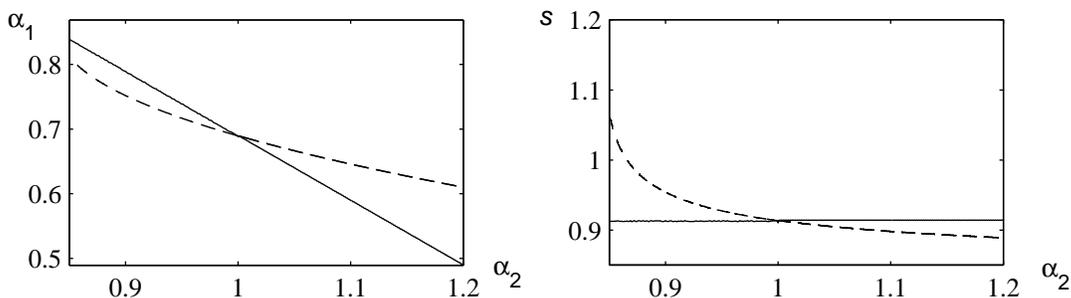
A longer lifetime of a design decreases the probability of success in quality improvements. It discourages quality research and encourages variety research. The right hand side of Figure 1 shows that the negative quality effect dominates with respect to total R&D effort. By showing only small deviations of *total* R&D with respect lifetime, however, the figure does not reveal the underlying intersectoral movements in opposite directions, which are comparatively large. For example, if the lifetime of a design is 20 years instead of 10, market employment in variety

<sup>4</sup>Note that  $\chi$  corresponds to  $1 - \xi$  in Stokey and to  $1 - \lambda$  in Jones and Williams.

research raises from 2.2 to 3.0 percent and employment in quality research decreases from 4.9 to 3.3 percent while optimal employment remains at 3.1 percent in both sectors. As a consequence of these counterbalancing intersectoral shifts the overall R&D ratio  $s^{SP}/s^{DC}$  changes only from 0.91 to 1.03 rendering a market employment in R&D very close to the socially optimal level. Generally we observe that changing the lifetime of a design has a smaller impact on total R&D effort as suggested by numerical studies of one-sector R&D models. Instead of discouraging (or encouraging) total R&D effort it mainly leads to a substitution from quality research to variety research with little impact on total resources devoted to R&D.

At first sight one might be tempted to think that the general two-sector R&D model will show a large sensitivity of results with respect knowledge spillovers because it establishes so many ways of inter- and intrasectoral knowledge flows. This is, however, not generally the case. The growth rates  $g_c$  and  $g_{TFP}$  (i.e. the sum of  $g_n$  and  $g_Q$ ) are predetermined by the data and tie down the endogenously obtained value for  $\alpha_1$ . Thus, any variation in one of the other knowledge spillovers will mainly cause an adjustment of  $\alpha_1$  with little impact on growth rates and economy-wide employment in R&D.

FIGURE 2. Knowledge Spillovers and R&D Shares



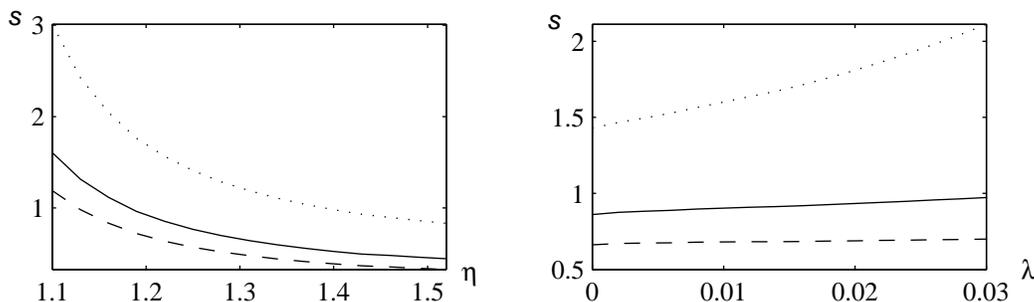
Solid lines: maintaining the assumption of symmetric R&D sectors:  $\beta_1 = \alpha_2$ ,  $\beta_2 = \alpha_1$ .

Dashed lines: maintaining the benchmark parameterization  $\beta_1 = 1$ ,  $\beta_2 = 0.7$ .

Figure 2 shows adjustment of  $\alpha_1$  and the implied change of the R&D ratio for varying  $\alpha_2$ . Results when the symmetry assumption between sectors continues to hold (i.e.  $\alpha_1 = \beta_2$ ,  $\alpha_2 = \beta_1$ ) are represented by solid lines. In that case inter- and intra-sectoral spillovers are almost linearly substituted against each other with virtually no impact on the overall  $s^{SP}/s^{DC}$  ratio. Dashed lines show results when the symmetry assumption is abandoned and  $\beta_1$  and  $\beta_2$  are fixed to

benchmark values. This scenario causes some variation in R&D shares; for parameter values within a reasonable range, however, the effect is still relatively small.<sup>5</sup>

FIGURE 3. R&D-Share: Social Planner/Market Solution ( $s$ )  
Markup Factor and Population Growth



Solid lines  $r^* = 0.07$ , dashed lines  $r^* = 0.1$ , and dotted lines  $r^* = 0.04$ . Other Parameters as in Table 1.

The lower the markup factor the lower the incentive to invest in R&D and hence the lower the market employment in research and the lower the  $s^{SP}/s^{DC}$  ratio. This is shown in Figure 3. Yet, changing markups within an empirically plausible range causes comparatively little variation of R&D shares around the socially optimal solution. For example, at the lowest value of 1.1 the R&D ratio is 1.60, less than half the size of the corresponding value in Jones and Williams (read off as 3.5 from the lower right panel in Figure 1, p. 77). Again, the explanation for the smaller sensitivity of results is that R&D is no longer the only engine of growth, and hence variation in market power, which affects R&D but not human capital accumulation, has a smaller impact on the R&D employment needed to generate the U.S. growth rate.

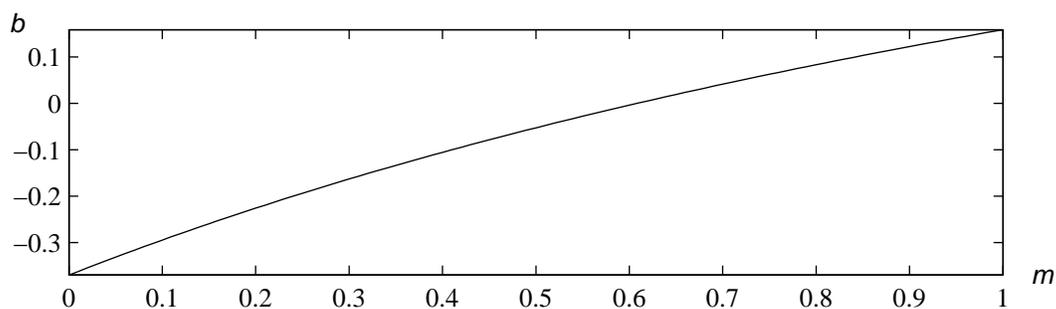
The righthand side of Figure 3 indicates that population growth has only little impact on the market's deviation from socially optimal R&D effort. This result is of course to a great extent explained by the fact population growth is not an externality. It does neither appear in (10a) nor in (12) and (13). This means that it affects factor allocation only very indirectly through the estimated value for the externality  $\alpha_1$  (in the calibration of growth rates (8) to (10)).

From a small effect of population growth on the market's misallocation of resources one cannot necessarily conclude an overall small economic impact. It might mainly affect growth rates of

<sup>5</sup>Note that growth rates become negative as  $\alpha_1$  approaches 0.7, the fixed benchmark value of  $\beta_2$ . Given that growth rates are predetermined at certain positive values by the data, the set of equations (8) to (10) is solvable only for  $\alpha_2$ 's sufficiently larger than 0.7.

human capital and research output which are identical for social planner and market. While an intermediate degree of altruism imposed in benchmark calibration ( $m = 1/2$ ) has indeed revealed a small correlation between population growth and economic growth the question occurs whether this result is robust against alternative assumptions about altruism. It is answered in the final numerical experiment.

FIGURE 4. The Impact of Altruism:  
Correlation between Population Growth and Economic Growth:  $b = \partial g_c / \partial \lambda$



Solid lines lines: recalibration of  $\theta$ , dashed lines: benchmark  $\theta = 2.4$ .

Figure 4 shows the estimated correlation,  $b$ , for altruism ranging from pure egoism ( $m = 0$ ) to pure altruism ( $m = 1$ ). If the U.S. were populated by households with Millian preferences population growth would have a measurable negative effect on economic growth with a correlation coefficient of about  $-0.37$ . For increasing altruism the correlation coefficient decreases in absolute terms and becomes positive when  $m$  surpasses 0.6. If households have Benthamite preferences (which are frequently assumed in endogenous growth literature) the correlation is about  $+0.16$ . Empirical studies, however, suggest that the degree of altruism towards future generations is clearly less than one and possibly small (See Altonji et al., 1997, Laitner and Ohlsson, 2001). For intermediate values of altruism the model provides the result that scale effect, dilution effect, and preference effect almost compensate each other so that the correlation of population growth and economic growth is close to zero. This result contrasts sharply with the earlier endogenous growth literature where population growth is the sole driving force of technological progress (and a correlation coefficient larger than one is obtained). Yet, it accords with the empirical evidence (see e.g. Brander and Dowrick, 1994).

An assessment of the importance of R&D, labor force growth, and human capital accumulation for economic growth can also be given by a growth accounting exercise. Using equation (9),

GDP growth,  $g_Y = g_c + \lambda$ , consists of TFP growth  $(g_n + g_Q)/(\sigma - 1)$ , and a second term  $g_H$ , which can be further subdivided into factor quantity growth  $\lambda$  and factor quality growth  $g_h = g_H - \lambda$ . Now consider the calibrated U.S. economy.<sup>6</sup> Without education, TFP growth would explain  $1 - \lambda/(g_c + \lambda) = 55\%$  of economic growth. With education, TFP growth explains  $1 - g_H/(g_c + \lambda) = 0.39\%$ , a much lower share which is also closer to empirical estimates. Factor quantity growth contributes  $\lambda/(g_c + \lambda) = 45\%$  to GDP growth and factor quality growth contributes 15%. The quality contribution corresponds roughly with the empirical estimates and is completely missing in R&D growth models without educational sector.<sup>7</sup>

## 5. CONCLUSION

This paper has investigated a two-R&D-sector growth model augmented by human capital accumulation and impure altruism. Results from a calibration with U.S. data have argued that the market share of employment in R&D is closer to the socially optimal level than suggested by earlier numerical studies. The deviation from optimal R&D is also less sensitive to parameter variation than previously obtained. The finding is explained by substitution possibilities between R&D activities and the additional role of human capital growth.

For a correct assessment of the result note, however, that the analysis has also shown that little deviation of the *economy-wide* R&D effort from the social optimum is compatible with relatively large *sectoral* deviations. Hence, the conclusion that laissez-faire provides approximately the optimal resource allocation, does not necessarily follow. Because of specific sectoral externalities a social planner might allocate researchers quite differently to sectors than the market although he chooses almost the same overall employment in research.

The investigation has also found that an explanation of last century's average U.S. growth by R&D-activities and human capital accumulation is compatible with the empirical finding of a small and probably negative correlation between population growth and economic growth.

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<sup>6</sup>Note the different focus of growth accounting. Here, we investigate GDP growth while the last section has focussed on growth of income per capita.

<sup>7</sup>Pencavel (1993) estimates that 23.4 percent of American economic growth during 1973 to 1984 was attributable to increased education. Dougherty and Jorgenson (1996) estimate that education has contributed 18.5 percent to U.S. growth between 1960 and 1989.

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