What Drives Labor Market Flows?*

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Abstract

I study an equilibrium model of the labor market with firm- and worker-level shocks and evaluate their relative contribution to the labor market flows. Firms hire and shed workers in response to firm-specific productivity shocks. Workers and firms learn about the quality of their employment match and separate when they realize they are mismatched. Crucially, match quality and productivity shocks must interact in order to explain the hazard rates of separation in the cross section of firm growth rates and workers’ tenures – a property I call technology adoption shocks. The model, calibrated to a large panel dataset of individual labor market histories in Austria, predicts that only 21%–30% of the separations are driven by firm-level shocks. This suggests that worker-level heterogeneity is crucial for explaining the magnitudes of worker mobility. Distinguishing different driving forces of the labor flows is important for evaluating effects of labor market policies.

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1 Introduction

In developed countries, between 4–15% of workers separate from their employers on average every quarter, and about the same number of workers are hired. This constitutes a large number of workers who change their employer or labor market status every quarter. What are the forces that drive the relocation of workers? One part of these worker flows can be attributed to job flows, as firms hit by productivity and other types of shocks expand and contract their workforce, thus creating and destroying job positions. Worker flows, however, exceed job flows by a wide margin, even at the establishment level. Establishments which increase their employment not only hire new workers, but also fire some of their current employees. Furthermore, new workers join shrinking establishments. This confirms that – in addition to the firm-level shocks – there are forces at the worker level which are important drivers in the decision to end a work relationship.

The goal of this paper is to explain the relationship between job and worker flows in the cross-section of firms, and to quantify the extent to which the large magnitude of worker flows is driven by firm-level shocks, match-level shocks, or search on the job. I develop a theoretical model of equilibrium labor market flows and assess it empirically using a large panel dataset. I use the model to estimate the relative importance of different driving forces behind match separations – match-level shocks associated with learning about the quality of the match between a firm and a worker, firm-specific productivity shocks, and search on the job. The model I develop is able to distinguish between these three sources of separation.

The new element of the model is to allow for interdependence between the firm- and match-level shocks. A better productivity shock does not make all workers in the firm automatically more productive. Some workers are not able to adopt the new technology, and even though they used to be well-suited for the job when the old technology was in place, the new technology makes them less valuable to the firm. I refer to shocks with this feature as technology adoption shocks.

I show that the ability of the model to match appropriate moments in the data on job flows, worker flows, and heterogeneity in the productivity at the firm and worker level plays a crucial role in the evaluation of the effects of different labor market policies. Omitting or incorrectly calibrating this heterogeneity may underestimate productivity losses generated by changes in the cross-sectional distribution of firm-worker matches in response to these policies by a significant amount.

The model economy is populated by workers who search for jobs and multiworker firms which post vacancies in a labor market with search frictions. Multiworker firms allow me
to model the dynamics of firm growth rates. Each match between a worker and a firm is characterized by an unobserved match quality. The worker-firm pair learns about the quality by observing a stream of output realizations. Matches that are sufficiently likely to be of a low quality end, and the worker becomes unemployed. Workers search for new jobs both when unemployed and employed, and choose to accept offers that improve their current position. Firms are exposed to idiosyncratic, firm-specific productivity shocks. These productivity shocks not only determine the productivity level of the firm but also impact the match quality of employed workers. This captures the idea that technological growth is partly embodied in workers — innovation makes some skills obsolete, and existing workers might not be able to operate the new technology. These technology adoption shocks can also be viewed as capturing the need of the firm to restructure its workforce in response to a productivity shock. Caliendo, Monte, and Rossi-Hansberg (2012) use data on French manufacturers to document that firms which grow tend to change their organizational structure.

The technology adoption shocks are crucial for understanding the empirical relationship between firm-specific shocks and the probability of separation from a job at a given tenure. Firms which experienced a positive productivity shock and grow actually have a higher separation rate than firms with zero employment growth. The higher separation rate translates into a higher risk of leaving a job for workers across all tenures. In growing firms, even workers with high tenures face an increased risk of match dissolution. In absence of technology-adoption shocks, workers with high tenures would have a negligible hazard rate of separation as they are believed to be the high quality workers. However, due to the technology-adoption shocks, the high-quality workers can turn into low-quality workers and eventually separate from a growing firm even at high tenures.

There are three main forces which generate worker flows. First, firms respond to the productivity shocks by adjusting their size. Following a positive productivity shock, the firm extends its workforce by hiring more workers, while it sheds the least productive workers in response to a negative shock. Second, even in absence of the firm-level productivity shocks, workers leave firms when they realize that the quality of their current match is likely to be low. Finally, workers can be attracted by an outside offer and leave the firm for a better job position. The data suggest that all motives are important but we need a structural model to assess their relative importance.

I use the model to evaluate the importance of these channels as a driving force of separations. The key for distinguishing separations due to firm-level or worker-level shocks in the model is the different nature of these shocks. The rules describing the decision to leave
the firm in the model are characterized by a set of thresholds for the perceived quality of the worker-firm match that are functions of the firm-level productivity. The worker-level shocks are modeled through innovations to worker’s perception of her match quality. The innovations arrive continuously as workers learn about the quality of their match by coming to work every day and completing tasks. On the other hand, the events which govern the firm-level productivity shocks happen rather infrequently, as they represent changes in the technology of the firm or its competitors, or changes in the demand for a good. Therefore, if a separation occurs because the perceived match quality crosses the boundary, I attribute this separation to match-level shocks; if it occurs because of a shift in the boundary, the separation is attributed to a firm-level shock.

I use a panel dataset of individuals from Austria to assess the model empirically. The data come from Austrian social security records and contain labor market histories of almost all individuals in Austria over the period 1986–2007. In particular, I observe individual employment and unemployment spells, with an exact begin and end date of each spell. Moreover, the record for employed individuals contains annual income, days worked and the identification code of the establishment. This structure allows me to merge individual workers into establishments while being able to trace the workers as they move between establishments. For each establishment I measure the job and worker flows, and analyze the structure of the pool of the separated workers. Specifically, I explore how the probability of being separated from a job at different tenures interacts with the establishment-level growth rate.

I calibrate the model to match aggregate labor market flows statistics, establishment-level employment dynamics and the average hazard rate profile. The model matches several important cross-sectional patterns. The joint cross-sectional distribution of job flows with the hiring and separation rates is as observed in the data. Growing firms exhibit a high separation rate for two reasons. First, even in growing firms, some workers learn that their match is poor and decide to separate. Second, growing firms are those who experienced positive productivity shocks and, through technology adoption, shed some workers who become unsuitable matches. Shrinking firms still exhibit a high hiring rate. A negative productivity shock makes the firm shed workers who are most likely to be bad matches, but it is still on average profitable for such a firm to hire a random worker from the pool of unemployed people.

The model correctly predicts the cross-sectional distribution of the hazard rate of separation at different worker tenures and firm growth rates. The most intriguing feature of
these separations is that all workers, including those with high tenures, face a higher hazard rate of separation in rapidly growing than in non-growing firms. The model generates such a pattern through technology adoption shocks. Growing firms have experienced an increase in their productivity. Every change in technology is associated with the risk that the workers’ match quality drops. This affects workers with long tenures who were believed to be good matches but now become less valuable to the firm. They eventually separate which translates into a higher risk of separations of high-tenure workers in growing firms.

Finally, I use the calibrated model to evaluate the contribution of worker- and firm-specific shocks to overall worker flows. In the full model, using the identification method described above, I find that about 40% of all separations are attributable to match-level shocks, about 30% to workers receiving a better outside offer, and the remaining 30% of separations are to firm-level productivity shocks.

1.1 Related literature

The paper builds on Jovanovic (1979) and Moscarini (2005) but extends the model to multiworker firms and introduces firm-level productivity shocks. Nagypal (2007) uses a similar framework to disentangle learning about the match quality from learning by doing. The theoretical model in this paper mainly differs from hers in that it allows for search on the job and the technology-adoption shocks.

The effect of the labor market policies has been studied in different contexts. Hopenhayn and Rogerson (1993) examine the effects of dismissal costs in a model of a firm which has decreasing returns to scale and decides about its employment level. Workers are homogenous and there are no search frictions, thus this model mainly speaks to effects of the labor market policies on firms’ employment, job creation and destruction. Dismissal costs lead to a productivity loss through the inability of the firm to equalize the marginal product with the marginal costs. Qualitatively, the dismissal costs equal to annual wages decrease employment by 2.5%. Pries and Rogerson (2005) examine effects of several labor market policies in a search model where a unit of analysis is a match between a firm and a worker. Each match is characterized by an unobserved match quality which is revealed with some probability. Low quality matches separate immediately while high quality matches survive.

The paper contributes to the strand of literature focused on understanding the sources of job turnover and labor market flows. Jovanovic and Moffitt (1990) use a model of sectoral mobility where worker flows are driven by both the sectoral demand shocks and the worker-employer mismatch. The model is then used to evaluate the relative importance of
the two channels for labor mobility. While their paper focuses on disentangling mismatch from sectoral shifts, this paper analyzes the relative importance of mismatch and idiosyncratic firm-level shocks. Despite their different focus, both papers find mismatch to be a significantly more important determinant of labor market flows.

The model developed in this paper distinguishes between job and worker flows, and thus relates to the literature which analyzes their joint behavior. Kiyotaki and Lagos (2007) and Pries and Rogerson (2005) both study an equilibrium model of a labor market characterized by job-to-job transitions and replacement hiring, which gives rise to a meaningful distinction between the job and worker flows. However, these models are based on one-to-one matches between a firm and a worker, and thus cannot distinguish the firm-level and match-level shocks, which is the focus of this paper.

The empirical distribution of the job and worker flows in the cross-section is studied in Davis, Faberman, and Haltiwanger (2006, 2012). Using establishment-level data for the U.S. the authors document a strong cross-sectional relationship between these flows, and show that it is important for understanding the aggregate movements in hires and layoffs over time. Faberman and Nagypal (2008) suggest a mechanism which can rationalize some features of the observed cross-sectional patterns, in particular, the relationship between quit and layoff rates, and the establishment growth rate.

Any search model which intends to account for the job-to-job transitions has to incorporate search on the job in some way. Nagypal (2005) argues that the basic job-ladder models generate job-to-job flows which are too low. She suggests that to overcome this feature, the value of the match cannot stay constant throughout the duration of the match, but should decrease in a way that does not induce the worker to leave her job. The model developed in this paper replicates the magnitude of the job-to-job flows through a mechanism consistent with Nagypal (2005). The value of a match is continuously updated because it reflects the information that the worker and employer have about the match quality. If bad news about the match arrives, the value of the match to the worker decreases, yet it can still be high enough so that the worker does not have a reason to join the unemployment pool.

2 Empirical motivation

I start by documenting pronounced patterns in the labor flows data and relate them to the firm- and worker-level shocks as driving forces of the worker flows. I argue that while the presented evidence is informative about the driving forces, it is not sufficient for inferring
their relative importance for the magnitude of the worker flows. The presented evidence will also inform the model and its calibration in the subsequent sections.

For the empirical analysis, I use the Austrian Social Security Database (ASSD) dataset, described in detail in Appendix C. The dataset contains labor market histories of almost all individuals over the period of 20 years together with the establishment identifier of their employers. The Austrian labor market is relatively flexible despite the existing regulations, and was not subject to substantial business cycle fluctuations during the recent decades. This stationarity, together with the richness and the structure of the dataset, makes the use of Austrian data particularly appealing for the purposes of this paper.

2.1 Job and worker flows

The extent of heterogeneity across establishments is typically measured by the excess job reallocation introduced in Davis, Haltiwanger, and Schuh (1998). The excess job reallocation equals the sum of the jobs that are created and destroyed beyond what is necessary to accommodate the net employment changes. The job creation and destruction at time \( t \) in industry \( s \) is defined as

\[
JC_{st} = \sum_{e \in S} \max (E_{et} - E_{e,t-1}, 0) \quad \text{and} \quad JD_{st} = -\sum_{e \in S} \min (E_{et} - E_{e,t-1}, 0),
\]

where \( E_{et} \) is employment in establishment \( e \) at time \( t \), and \( S \) is the set of all establishments in industry \( s \). The excess job reallocation is the difference between the sum of the job reallocation and the net employment growth,

\[
ER_{st} = JC_{st} + JD_{st} - |JC_{st} - JD_{st}|.
\]

To express excess reallocation in rates, I divide it by the gross job reallocation,

\[
R_{st} = JC_{st} + JD_{st},
\]

\[
er_{st} = \frac{ER_{st}}{R_{st}} = 1 - \frac{|JC_{st} - JD_{st}|}{JC_{st} + JD_{st}} \in [0, 1].
\]

The excess reallocation rate is zero if and only if either the job creation or the job destruction equals zero. A positive excess reallocation rate indicates that the industry exhibits simultaneous creation and destruction of jobs.

Table 1 reports the quarterly job creation and destruction rates as a share of employment, and excess job reallocation rate for 13 industries, averaged over the period 1986–2007. The job creation and destruction rates are defined as

\[
jc_{st} = JC_{st}/0.5 (E_{st-1} + E_{st}), \quad jd_{st} = JD_{st}/0.5 (E_{st-1} + E_{st}).
\]

Despite a lot of heterogeneity in magnitudes of the job creation and destruction rates, the excess reallocation rate is uniformly high in all industries. Even though the magnitude can depend on the level of industry disaggregation, Figure 1 shows that pattern does not change when I use a finer industry division. In particular, for Figure 1 I split the establishments using their 4-digit industry code and calculate job cre-
<table>
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<th>$jc$</th>
<th>$jd$</th>
<th>$er$</th>
</tr>
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<td>accommodation and food services</td>
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<td>0.12</td>
<td>0.73</td>
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<td>transportation</td>
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<td>0.90</td>
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<tr>
<td>finance and business services</td>
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<td>0.04</td>
<td>0.69</td>
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<tr>
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<td>0.03</td>
<td>0.03</td>
<td>0.85</td>
</tr>
<tr>
<td>other services</td>
<td>0.05</td>
<td>0.05</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Table 1: Job flows by industry, average quarterly rates 1986–2007. The table shows the quarterly average job creation, destruction and excess reallocation rates for different industries over the period 1986–2007. The job creation and destruction are expressed as shares of the industry-wide employment, while the excess reallocation as a share of the gross job reallocation ($jc + jd$). Source: Austrian social security data.

I sort the industries into bins according to their job reallocation rate ($jc_{st} + jd_{st}$) and compute the average excess job reallocation rate for each bin. The high magnitude of the excess reallocation rate indicates that even within narrowly defined industries, shrinking establishments coexist with growing establishments. This fact is often interpreted as evidence that the idiosyncratic component of the labor demand is an important determinant of the magnitude of the worker flows.

However, this interpretation is not unambiguous. A model with worker-level shocks but no role for the firm-level shocks can generate the same pattern. Suppose that workers leave their firm following an idiosyncratic shock (at the worker level), and that a firm is not able to immediately replace these workers. In such a model, shrinking and growing firms can coexist: some firms shrink because workers left and the firm could not replace them quickly, while other firms grow due to replacement hiring. The high magnitude of the excess job reallocation rate could thus also be driven by the worker-level shocks.

So far, I documented that the job flows are large. Yet the worker flows behind these job flows are even larger. I describe them using the establishment-level hiring and separation rates, and relate them to the job flows. I follow the Davis, Haltiwanger, and Schuh (1998) methodology and define the growth rate of an establishment $e$ at time $t$ as
Figure 1: Excess reallocation rate in 4-digit industries. The figure shows the average quarterly excess reallocation rate, defined as the ratio of the excess reallocation and job reallocation, in 4-digit industries over the years 1986–2007. I sort the quarterly observations for 4-digit industries into 100 bins based on their reallocation rate and plot the mean excess reallocation rate in each bin. Source: Austrian social security data.

\[ g_{et} = \frac{(E_{e,t} - E_{e,t-1})}{Z_{et}}, \] where \( E_{e,t} \) is the number of employees and \( Z_{et} = 0.5 (E_{e,t} + E_{e,t-1}) \) is the measure of the employer size. I similarly define measures of establishment-level hiring and separation rates to be \( h_{et} = \frac{H_{et}}{Z_{et}} \) and \( s_{et} = \frac{S_{et}}{Z_{et}} \) where \( H_{et} \) and \( S_{et} \) is the number of hired and separated workers in establishment \( e \) at time \( t \). To analyze the relationship between worker and job flows at the establishment level, I sort establishments into narrow bins based on their growth rate and calculate the employment-weighted hiring and separation rates for each bin. This way I allow for a non-linear relationship between these flows. The relationship is plotted in Figure 2.

Several interesting patterns stand out. First, hiring and separations occur simultaneously both in growing and shrinking establishments. Workers join shrinking establishments while other workers leave establishments which do well and grow. Second, there is a lot of worker turnover even at establishments whose net employment change is zero. This suggests that there is ample space for the worker-level shocks.

The estimated relationship can potentially be driven by the composition of establishments in each growth bin. For example, the pooled regression may ignore unobserved heterogeneity in the volatility of firm-level shocks. More volatile establishments may have different mean hiring and separation rates and, at the same time, their observed growth rates would be mechanically sorted into both tails of the growth rate distribution. I thus estimate this relationship with establishment fixed effects\(^1\) which absorb the heterogeneity in mean hiring.

\(^1\)I run a regression of the hiring and separation rates on the full set of dummy variables for the growth
Figure 2: Empirical hiring and separation rates as functions of the firm growth rate. I follow the Davis, Haltiwanger, and Schuh (1998) methodology and define the growth rate of an establishment $e$ at time $t$ as $g_{et} = (E_{e,t} - E_{e,t-1})/Z_{et}$, where $E_{e,t}$ is the number of employees and $Z_{et} = 0.5(E_{e,t} + E_{e,t-1})$ is the measure of the employer size. I similarly define measures of establishment-level hiring and separation rates to be $h_{et} = H_{et}/Z_{et}$ and $s_{et} = S_{et}/Z_{et}$ where $H_{et}$ and $S_{et}$ is the number of hired and separated workers in establishment $e$ at time $t$. I control for the establishment fixed effects. The gray bars indicate the distribution of employment across growth-rate bins. Source: Austrian social security data.

and separation rates and isolate the within-establishment variation. Indeed, as the dotted lines in Figure 2 show, heterogeneity exists and partly affects the cross-sectional pattern, mainly the hiring rate in the shrinking and separation rate in the growing establishments. Nevertheless, the cross-sectional pattern remains similar and thus I will ignore this type of heterogeneity and use only the idiosyncratic shocks at the establishment level in the model.

While the difference between the job and worker flows suggests that there is ample space for the worker-level shocks, a model with pure firm-level shocks could generate such a pattern through time-aggregation. The job flows are measured as a change in employment between two points in time while the worker flows add up the absolute values of all employment changes within the period. This itself can generate higher worker than job flows if a firm increases and decreases its employment more than once between the two measurement dates. Proper time aggregation and a suitable choice of the model frequency is therefore crucial for rate bins, where I also include establishment fixed effects. The figure then depicts the coefficients on the dummy variables.
The presented evidence suggests that both the firm- and worker-level shocks stand behind the worker flows but it does not inform us on how important these shocks are. I construct a model which has a role for both firm- and worker-level shocks and use it to evaluate their relative importance.

### 2.2 Tenure length

The worker- and firm-level shocks can be correlated. To understand this correlation, I look at the cross-sectional relationship of the firm growth rate and the hazard rate of separation at different tenures. I view the firm growth rate as a proxy for the firm-level shock while the tenure is an important determinant of worker’s separation risk. Workers who differ only in their tenure have very different probabilities of separating from the employer as is illustrated\(^2\) in Figure 3.

The separation risk across tenures interacts with the firm growth rate. Figure 4 shows the hazard rate curves in establishments with different growth rates. Although one might expect that in growing firms, the hazard rate of separation will be the same or lower than the hazard rate in firms which do not grow, I find that the entire hazard rate curve shifts up with the establishment growth rate. Thus, comparing workers with the same tenure, a worker is

\(^2\)There is a spike in the hazard rate of separation at 3 years of tenure, which is the time when a worker under the old social security system becomes available for the severance payment.
Figure 4: Empirical hazard rate of separation as a function of tenure and establishment growth rate. The figure shows the quarterly hazard rate of separation in growing firms (top panel) and shrinking firms (bottom panel). Separation as a function of tenure conditional on their growth rates (bottom two panels). The firm growth rate is quarterly. For each range of the establishment growth rate $g$, the hazard rate is calculated as the share of workers at a given tenure who separate relative to all workers at that tenure, in firms with the given growth rate. Source: Austrian social security data.

more likely to separate in a rapidly growing firm than she is in a firm with zero growth rate. The hazard curve also moves up with the contraction rate in the shrinking firms. The shift is almost parallel which indicates that in relative terms the high-tenure workers are affected the most.

Notice that these results are not implied by the fact that growing establishments have a higher average separation rate. The increase in the average separation rates could be driven by the composition of workers, as fast-growing firms have a higher share of low-tenured workers who face the highest hazard rate of separation.

The inclusion of the establishment fixed effects only strengthens these results, as Fig-
ure 5 documents, and thus the firm heterogeneity does not drive the pattern. This provides strong evidence that the firm- and worker-level shocks indeed interact. After receiving a new productivity shock, a firm may need to restructure its workforce as some workers will not be able to cope with the new technology. Thus, even though workers with high tenures are typically thought of as high-quality workers with small hazard rate of separation, their risk of separation increases in firms which receive a new technology shock. This is true even in firms which grow in response to a positive shock.

I capture this mechanisms through the technology adoption shocks. Firm-level shocks, both positive and negative, not only change the productivity of the firm but also make some skills obsolete. Some workers will not be able to adopt the new technology and even if they used to be valuable to the firm when the old technology was in place, now their value to the firm drops. In the model, every match between a firm and a worker is characterized by a match quality. Every change in firm’s technology, both positive and negative, is associated with a certain probability that the individual match quality changes from high to low.

3 Model

The economy is populated by a continuum of risk-neutral workers who search for jobs and a continuum of firms of measure $K$ that post vacancies in a labor market with search frictions. Workers and firms are matched to produce output. While being matched, the firm-worker pair updates their beliefs about the quality of the match using the time series of observations of the output flow produced in the match. The match is terminated if the surplus from the match falls to zero, if the worker leaves to accept a better job, or for exogenous reasons at the rate $\delta$.

3.1 Production technology and shocks

Firms have a constant returns to scale production technology with labor as the only input, and employ many workers. Firm $k$ is characterized by its time-varying productivity level $\hat{A}_k(t)$. Firms are endowed with a linear production technology, implying that there are no diminishing marginal returns to labor at the firm level and thus newly hired workers do not crowd out old hires.

The output of worker $l$ employed in firm $k$ depends both on the firm’s productivity $\hat{A}_k(t)$ and on the match-specific productivity $\mu_{lk}(t)$ that measures the degree of suitability of the worker for the particular technology in place in the given firm. The worker produces the
output flow

\[ dy_{lk}(t) = \hat{A}_k(t) \, dt + dx_{lk}(t), \]  

where \( x_{lk}(t) \) follows the diffusion

\[ dx_{lk}(t) = \mu_{lk}(t)dt + \sigma_x dW_{lk}(t) \]  

with an unobservable time-varying match-specific drift \( \mu_{lk}(t) \). Here, the Brownian motion increments \( dW_{lk}(t) \) are independent across workers, firms and time.

The value of \( \hat{A}_k(t) \) is specific to the firm while \( \mu_{lk}(t) \) is specific to the match between the worker \( l \) and the firm \( k \). The firm-worker pair observes the flow \( dy_{lk}(t) \) and the firm-
specific productivity shock \( \hat{A}_k(t) \), from which they can deduce the match-specific component \( dx_{lk}(t) \). However, the value of the match-specific productivity \( \mu_{lk}(t) \) is unobserved because the shock \( dW_{lk}(t) \) generates noise around the true value of \( \mu_{lk}(t) \). The worker–firm pair infers the value of \( \mu_{lk}(t) \) by solving a filtering problem, described in Section 3.2.

The firm-specific productivity \( \hat{A}_k(t) \) follows a finite-state Markov process with \( \hat{A}_k(t) \in \mathbb{A} \equiv \{A_1, \ldots, A_I\} \) and intensity matrix \( \Omega \).

Every time a firm and a worker meet, they first draw a signal \( p_0 \) about the match quality from the distribution \( G_0(p) \) with density \( g_0(p) \). This information is observed by both the firm and the worker and is used to decide whether or not to form a match. After they create a match, they draw the actual (unobserved) match-specific quality from the distribution \( G_{\mu}(:, p_0) \). I assume that the match quality \( \mu_{lk}(t) \) can attain 2 values, \( \mu_H > \mu_L \), and the signal \( p_0 \) determines the probability that the initial match quality is high.

The match quality \( \mu_{lk}(t) \) is not fixed throughout the duration of the match, and its changes are linked to changes in the firm-specific productivity \( \hat{A}_k \). Upon arrival of a new firm productivity shock, a high quality match can turn into a low quality match with probability \( \gamma \). Since \( \mu \) is unobservable, the switch from the high to low quality is unobservable as well. This is the technology adoption shock: when the new technology arrives, not all workers are able to adopt it. Some of workers’ abilities become obsolete for the new technology and thus matches which could have been good before the change become less valuable.

### 3.2 Evolution of beliefs

In this section I drop the subscripts \( lk \) to simplify the notation.

The value of the match \( \mu(t) \) can attain only 2 values and thus \( p(t) \), the probability that \( \mu(t) = \mu_H \), is a sufficient statistic for the learning problem. The belief \( p(t) \) is updated in two cases — after observing a new output realization \( dx(t) \) and after observing a change in the firm productivity level from \( A_i \) to \( A_j, i \neq j \). In the first case, the standard Wonham (1964) result implies that \( p(t) \) follows a diffusion process

\[
dp(t) = \sigma_p(p(t)) d\bar{W}(t)
\]  

---

3 The intensity matrix \( \Omega \) defines the transition probabilities across states \( \{A_1, \ldots, A_I\} \). For any \( \tau \geq 0 \), \( P_\tau = \exp(\tau \Omega) \) where \( \exp(\cdot) \) is the matrix exponential defines the transition matrix \( P_\tau \) such that \( \Pr(A_k(t+\tau) = A_j \mid A_k(t) = A_i) = P_\tau(i, j) \).
where

\[
\sigma_p(p) = p(1-p)s \quad \text{(4)}
\]

\[
s = \frac{\mu_H - \mu_L}{\sigma_x}
\]

\[
d\bar{W}(t) = \frac{1}{\sigma_x} (dx(t) - (\mu_H p(t) + \mu_L (1-p(t)))dt).
\]

Here, \(d\bar{W}(t)\) is a standard Brownian motion under the information set of the worker and the firm. The innovation process \(d\bar{W}(t)\) is the normalized difference between the realized output \(dx(t)\) and the expected output flow \((\mu_H p(t) + \mu_L (1-p(t)))dt\).

The belief about the match quality of the high and low type will, in expectation, drift to one and zero, respectively. Given that the model is calibrated so that workers with a sufficiently low \(p(t)\) separate, the average match quality of the workforce increases with tenure. Absent technology adoption shocks or an exogenous of separations, long-tenured workforce would consist primarily of high-quality matches with negligible hazard rates of separation. Even with an exogenous source of separations, the model could not match the increasing pattern of separations in the firm growth rates for growing firms across all tenures, documented in Figures 4 and 5.

The technology adoption shocks that augment the match quality of individual workers counteract this force. When the firm receives a new productivity shock, the belief is updated using the Bayes formula. Given the prior belief \(p\), the posterior belief \(p'\) is given by

\[
p' = p \Pr [\mu' = \mu_H | \mu = \mu_H] + (1-p) \Pr [\mu' = \mu_H | \mu = \mu_L]
\]

\[
= p (1 - \gamma)
\]

The probability that a high-quality match is not affected by the change in the productivity is \(1 - \gamma = \Pr [\mu' = \mu_H | \mu = \mu_H]\). Since the technology adoption shock does not turn the low-quality matches into high-quality ones, it holds that \(\Pr [\mu' = \mu_H | \mu = \mu_L] = 0\).

Notice that the posterior belief \(p'\) is lower than the prior belief for all \(p\). Thus, a productivity shock, whether good or bad, makes the perceived match quality worse, and the firm-worker pair has to learn again whether the worker is suitable for the particular technology in place. The ranking of workers in the firm by their expected productivities does not change: better matches remain to be perceived as better. However, the expected productivity of workers who switched from high- to low-quality matches now starts drifting downward over time.
3.3 Wage setting

The wage-setting protocol is given by sequential auctions as in Postel-Vinay and Robin (2002) or Lise and Robin (2013). Firms make type- and state-contingent offers and counter-offers to workers. When an unemployed worker receives an offer, the wage is chosen so as to set the value of the match for the worker to the value of unemployment. The wage then remains constant until the worker receives an outside offer or the value of the match for the firm becomes negative.

If the worker receives an outside offer, the incumbent and poaching firm engage in Bertrand competition. The firm with the higher surplus attracts the worker, and the worker’s wage is determined by the second highest surplus.

This wage-setting mechanism implies that the triplet \((w, A_i, p)\) describes the state for the worker-firm pair. The law of motion for the pair \((A_i, p)\) has been described previously, while the current wage rate \(w\) is the sufficient statistic for the history of wage increases and cuts for the given worker.

Let \(W(w, A_i, p)\) and \(J(w, A_i, p)\) be the worker’s and firm’s value of being in a match. Let \(B\) be the value of being unemployed and define the surplus of the match

\[
S(A_i, p) \equiv W(w, A_i, p) - B + J(w, A_i, p).
\]

Here the outside value of a vacancy is zero because the firms can create jobs at no costs. However, there are flow costs of advertising vacancies, which will affect firm’s decision about how many vacancies to post. Notice that the surplus of the match does not depend on the wage rate \(w\), since the wage only redistributes surplus between the firm and the worker.

Each job offer is characterized by the productivity level of the offering firm, the prior belief \(p_0\), and the wage rate \(w\). Unemployed workers are hired at a wage \(w\) which makes them indifferent between accepting the offer or not,

\[
W(w, A_i, p_0) = B.
\]

If an employed worker of type \((w, A_i, p)\) meets a firm of type \((A_j, p_0)\), she takes the offer only if \(S(A_j, p_0) > S(A_i, p)\), and the wage \(w'\) in the new firm is set to

\[
W(w', A_j, p_0) - B = S(A_i, p).
\]

If it is the case that \(W(w, A_i, p) - B < S(A_j, p_0) < S(A_i, p)\), then the worker stays with
the current firm but is able to negotiate a wage increase up to the point where

\[ W(w, A_i, p) - B = S(A_j, p_0). \]

Finally, if \( S(A_j, p_0) < W(w, A_i, p) - B \), the worker stays in the firm with no wage change. If it is necessary, the wage is renegotiated after a new productivity shock or update of the beliefs. This is the case when the match surplus is positive (otherwise they would separate) but either the worker’s or the firm’s surplus are negative. In the first case,

\[ W(w, A_i, p) - B < 0, \]

and the wage is reset to \( w' \) such that \( W(w', A_i, p) - B = 0 \). If the wage is too high so that the firm’s surplus is negative,

\[ W(w, A_i, p) - B > S(A_i, p), \]

then the wage is changed to \( w' \) which makes the firm just indifferent between separating or not,

\[ W(w', A_i, p) - B = S(A_i, p). \]

### 3.4 Value functions

The assumption of the constant returns to scale technology allows me to study a match between a firm and a worker without keeping track of other workers employed in the firm. A worker can be either employed or unemployed. Both employed and unemployed workers are contacted by firms and choose to accept offers that improve their current position.

Unemployed workers consume their unemployment benefits \( b \) and are contacted by firms at the rate \( f_u \). Employed workers consume their wage \( w \) and are contacted by firms at the rate \( f_e = \psi f_u \), where \( \psi > 0 \) is a parameter that determines the relative search intensity of employed and unemployed workers.

As a consequence of the wage-setting mechanism, the value \( B \) of being unemployed is simply given by

\[ rB = b. \quad (7) \]

Normally, the flow value of unemployment, \( rB \), includes not only the flow of unemployment benefits \( b \) but also the expected flow gain from the arrivals of new job opportunities. In this
wage-setting mechanism, the first employer extracts the whole surplus from the worker, and thus this expected gain is zero.

Let $C$ be the ‘continuation region’ — the set of all $(A_i, p)$ for which an existing match is preserved and the worker does not leave into unemployment. This region is chosen optimally by the worker-firm pair. In Section 3.6, I show that the region is characterized by a set of separation thresholds.

Once the worker finds a match, the total surplus generated by the match satisfies

$$rS(A_i, p) = y(A_i, p) - b + \frac{1}{2}\sigma^2_p(p) \frac{\partial^2 S(A_i, p)}{\partial p^2} - \delta S(A_i, p) +$$

$$+ \sum_{j \neq i} \omega_{ij} (S(A_j, p') - S(A_i, p)) \quad \text{if} \quad (A_i, p) \in C$$

$$S(A_i, p) = 0 \quad \text{if} \quad (A_i, p) \in C^C$$

where $p'$ is given in (5), $\omega_{ij}$ are elements of the intensity matrix $\Omega$, and $y(A_i, p) = A_i + p\mu_H + (1 - p)\mu_L$ is the expected output flow from the match. The value of preserving the match equals the net flow return $y(A_i, p) - b$, the gain from learning reflected in the second-derivative term, the expected loss from exogenous separations, $\delta S(A_i, p)$, and the net benefit from receiving a new productivity shock $A_j$.

The fact that the benefit from searching while on the job does not show up in the value function is again a consequence of the wage-setting rule. If the poaching firm can generate a higher surplus than the current firm, $S(A_j, p_0) > S(A_i, p)$, then the worker accepts the offer, and the Bertrand competition implies that the poaching firm sets the worker’s wage so as to set the value of the match for the worker to $S(A_i, p)$. The incumbent firm loses the match but the total surplus of the worker–incumbent firm pair remains unchanged. More specifically, the worker gains $[S(A_i, p) - (W(w, A_j, p_0) - B)]$ while the firm loses $J(w, A_i, p)$, which sums up to zero by equation (6).

### 3.4.1 Distribution of beliefs and firms’ productivities

Let $g(A_i, p)$ be the measure of workers with belief $p$ employed in firms with productivity $A_i$. This measure integrates to the total number of employed workers $E$. For $(A_i, p)$ such that a match does not exist, the measure is zero, $g(A_i, p) = 0$. For other $(A_i, p)$, the Kolmogorov forward equation describes the dynamics of this measure. Imposing that the distribution is
time-invariant, \( dg(A_i, p) / dt = 0 \), gives:

\[
0 = \frac{d^2}{dp^2} \left( \frac{1}{2} \sigma_p^2(p) g(A_i, p) \right) + \sum_{j \neq i} \omega_{ji} \frac{g(A_j, \frac{p}{1-\gamma})}{1-\gamma} \\
+ f_e \bar{v}_i g_1(p) \left[ \sum_j \int_{\tilde{p} : S(A_i, p) > S(A_j, \tilde{p})} g(A_j, \tilde{p}) d\tilde{p} \right] + f_u \bar{v}_i g_0(p) \left[ 1 - \sum_j \int_0^1 g(A_j, \tilde{p}) d\tilde{p} \right] \\
- \left[ f_e \sum_j \bar{v}_j \int_{p : S(A_j, p_0) > S(A_i, p)} g_0(p_0) dp_0 \right] g(A_i, p) - \sum_{j \neq i} \omega_{ij} g(A_i, p) - \delta g(A_i, p)
\] (9)

Each term in (9) describes one channel that contributes to the evolution of the measure \( g(A_i, p) \). The first term balances all flows that are due to learning. The measure at \((A_i, p)\) gains workers employed in firms with productivity \( A_j \neq A_i \) which received a new productivity shock \( A_i \) and have updated their beliefs using (5), as captured by the second term.\(^4\) Workers can flow to \((A_i, p)\) from employment or unemployment (the second line), in both cases conditional on the fact that they draw the prior belief \( p_0 \). Finally, the last line captures the attrition in the measure at \((A_i, p)\) due to search on the job, exogenous separations, and outflows caused by firms receiving new productivity shocks.

### 3.4.2 Vacancy posting and matching

To hire new workers, firms have to post vacancies. This decision depends on the expected value from randomly meeting a worker. Let \( L \) denote effective searchers. These include unemployed workers \( U \) as well as employed workers \( E \equiv 1 - U \). Since these workers search with different intensities, they must be appropriately weighted, \( L = U + \psi E \). For a firm with productivity \( A_i \), the expected value of meeting a worker is

\[
\bar{J}(A_i) = \frac{U}{L} \int \bar{S}(A_i, p_0) g_0(p_0) dp_0 \\
+ \frac{\psi E}{L} \sum_{j=1}^I \int \int (\bar{S}(A_i, p_0) - \bar{S}(A_j, p) + \frac{g(A_j, p)}{E} dp_0) g_0(p_0) dp_0,
\]

where \( x^+ \equiv \max\{0, x\} \). A randomly chosen worker is unemployed with probability \( U/L \) in which case a firm keeps the whole surplus. A randomly chosen worker is employed with probability \( \psi E / L \), and with probability \( \frac{1}{E} g(A_j, p) \) she is currently in a match with pro-

\(^4\)For convenience I also define \( g(A_i, p) = 0 \) for \( p > 1 \) to deal with the fact that \( p / (1-\gamma) \) can be greater than 1.
ductivity $A_j$ and belief $p$. A worker accepts an offer from a poaching firm if it provides a higher surplus. The poaching firm offers a wage which gives the worker the surplus from her current match, thus the gain for the poaching firm is the difference in the surpluses, $(S (A_i, p_0) - S (A_j, p))$.

The costs of posting $v$ vacancies is $C (v) = c_0 \frac{v^{1/c_1}}{1+c_1}, c_1 > 1, c_0 > 0$. The firm chooses the number of vacancies $v \geq 0$ to maximize the expected return net of posting cost,

$$\max_{v \geq 0} [v q \bar{J} (A_i) - C (v)],$$

where $q$ is the rate at which a vacancy meets a worker that is determined in equilibrium. The first-order condition yields

$$c_0 v^{c_1} = q \bar{J} (A_i) \text{ if } v > 0. \quad (11)$$

As long as the expected value from meeting a worker is positive, the firm posts vacancies up to the point where the marginal cost of posting a vacancy equals the expected value from filling the vacancy. The number of posted vacancies, $v (A_i)$, only depends on the firm’s productivity level $A_i$.

The decisions of individual firms determine the distribution of vacancies. Let $\bar{\omega}_i$ be the share of firms with productivity $A_i$ at time $t$. The total number of vacancies $V$ and the vacancy distribution $\bar{v}_i$ are

$$V = K \sum_{i=1}^{I} \bar{\omega}_i v (A_i), \quad \bar{v}_i = \frac{K \bar{\omega}_i v (A_i)}{V}. \quad (12)$$

Here $\bar{v}_i$ is the probability that a randomly chosen vacancy comes from a firm with productivity $A_i$.

Finally, the aggregate matching function determines the flow of matches formed at every instant as a function of the number of searching workers $L$ and the aggregate number of vacancies $V$,

$$M = M (L, V).$$

Workers and firms that become matched are randomly drawn from the pool of searchers and vacancies, and hence the vacancy meeting rate for the unemployed workers, $f_u$, and the rate
at which a vacancy meets a worker are given by
\[
    f_u = \frac{M(L, V)}{L}, \quad q = \frac{M(L, V)}{V}. \tag{13}
\]

I assume that the matching function is of the form \( M(S, V) = M_0 L^\alpha V^{1-\alpha} \) where \( M_0 > 0 \) and \( \alpha \in [0, 1] \). Define the vacancy-searchers ratio \( \theta = V/L \), then
\[
    f_u = M_0 \theta^{-\alpha}, \quad q = M_0 \theta^{1-\alpha}. \tag{14}
\]

### 3.5 Steady state equilibrium

I focus on the steady state equilibrium where aggregate variables are constant over time. The equilibrium objects \( f_u, f_e, q, L, V \) are determined by the joint distribution of beliefs \( p \) and firms’ productivity levels \( A \) in the economy which I described before.

The measure of workers’ beliefs and firms’ productivities determines the unemployment rate:
\[
    U = 1 - \mathbb{E} = 1 - \sum_{i=1}^I \int_0^1 g(A_i, p) \, dp. \tag{15}
\]

#### 3.5.1 Definition of the steady state equilibrium

A **steady state equilibrium** is a value function \( S(A_i, p) \) for the match surplus, a vacancy posting function \( v(A_i) \), distributions of vacancies and workers \( \{\bar{v}_i\}_{i=1}^I \) and \( g(A_i, p) \), and numbers \( \{\theta, f_u, f_e, q\} \) such that:

1. The surplus \( S(A_i, p) \) solves (8).
2. Given \( f_u, f_e, \{\bar{v}_i\}_{i=1}^I \), the measure \( g(A_i, p) \) solves (9) and is time-invariant.
3. The variables \( f_u, f_e, q, \theta, \{\bar{v}_i\}_{i=1}^I \) solve (12) and (14).

### 3.6 Decision rules and shock identification

#### 3.6.1 Decision rules

The decisions of workers are characterized by a set of separation thresholds. One set of the thresholds describes workers’ decision to quit to unemployment which determines the continuation region \( C \); another one the decision to accept an outside offer. The thresholds
depend on the productivities of the current and contacting firms, and the prior belief about the match quality.

A worker can, at any time, quit to unemployment. She chooses to exercise this option when the return from quitting exceeds the return from staying in the job. The indifference between staying in a firm with productivity $A_i$ and quitting to unemployment determines the separation threshold $p(A_i)$,

$$S(A_i, p(A_i)) = 0$$

such that the worker quits if $p \leq p(A_i)$. Therefore, $C = \{(A_i, p) : p > p(A_i)\}$.

If the value function is increasing in $p$ and $A_i$, then $p(A_i)$ is decreasing in $A_i$. A worker is thus more willing to tolerate an unpromising match in a firm with a higher productivity level. Conditional on $p$, a higher productivity increases her surplus from the match but does not affect the value of being unemployed. A firm is willing to keep the worker because existing workers do not crowd out new hires due to the linear production technology.

Workers receive offers from other firms while employed and decide whether to accept them or not. Once a worker meets a new firm, she draws a new prior belief $p_0$ and the match quality is drawn again from the distribution $G_\mu(\cdot; p_0)$. A worker prefers to accept the outside offer when her prospects on the current job are bad, be it because of the low perceived match quality itself or low firm productivity. The worker’s decision to accept an offer from firm with productivity $A_j$ when such an offer arrives is described by a separation threshold $\bar{p}(A_i, A_j, p_0)$ at which a worker is indifferent between staying in the firm with productivity $A_i$ and leaving to a firm with productivity $A_j$ after receiving a signal $p_0$ about the match quality in the prospective job,

$$S(A_j, p_0) = S(A_i, \bar{p}(A_i, A_j, p_0)).$$

The threshold $\bar{p}(A_i, A_j, p_0)$ is decreasing in the current firm’s productivity $A_i$ and increasing in the contacting firm’s productivity $A_j$, as long as the value function is increasing in $p$ and $A_i$ (see Appendix A.1) and is increasing in $p_0$.

### 3.6.2 Identification of separation shocks

A worker decides to separate from a firm if her belief happens to be below or at the separation threshold. The belief is updated continuously with the arrival of new signals about the match quality. The threshold, however, shifts up and down discontinuously after each change in
firm’s technology. The fact that the belief evolves continuously and the threshold moves discretely gives a clear notion of which force leads to the separation. Namely, if the belief hits the separation boundary, I attribute the separation to the worker-level shock. If the separation occurs due to the shift in the threshold, I attribute it to the firm-level shock.

The different nature of the evolution of the beliefs and the thresholds is crucial for distinguishing the shocks. The idea is that workers learn about the quality of their jobs by coming to work and completing daily tasks. By exercising the tasks the worker realizes if she is suited for her current job. The nature of the firm-level shocks is very different, though. The technology shocks in the model stand for multiple types of shocks — new innovation to the firm’s own technology or to technology of its competitors, or changes in the demand for the firm’s products. These are all events which occur infrequently rather than continuously, which explains the different nature of the firm-level productivity shocks.

Figure 6 illustrates this mechanism. It shows several different sample paths for the belief as a function of tenure, which is depicted on the horizontal axis. All paths start at the initial belief $p_0$. The red step function represents the separation threshold into unemployment.

Figure 6: Sample paths of beliefs implied by the model, with classification of the different types of separation.
Every change in the separation threshold represents the arrival of a new productivity shock. Besides shifting the threshold, all beliefs are also updated downward by a fraction \( (1 - \gamma) \) through the technology-adoption shocks.

Workers whose beliefs follow paths depicted in blue eventually separate to unemployment. Workers with belief paths depicted in blue that hit the separation threshold in its horizontal parts, separate for reasons attributed to the worker-level shocks. The blue sample path that crosses the separation threshold in its vertical part is classified as a layoff due to the firm-level shocks. This is because the change in the firm-level technology moved the separation threshold above the worker’s current belief.

The threshold shifts down in firms which experience a positive technology shock and thus absent the technology adoption shocks, this shift would result in no immediate separation. Due to the technology adoption shocks, the belief is updated downward as well, and can move low enough to result in a separation. In simulations, such a separation is also attributed to firm-level shocks.

Finally, the green sample path represents a worker who accepts an offer from an outside firm, while the purple one stands for a worker who does not separate during the observed time period. The beliefs of these workers trended upward and most of the time were in the region where workers would have rejected any offer from outside firms. After a technology shock experienced by a firm, the beliefs are updated downward and eventually reach the area where workers are willing to accept outside offers, conditional on receiving one. One of these workers in the figure receives an offer she is willing to accept. This illustrates that in this model, separations to other jobs are different in nature than separations to unemployment, and thus I treat them separately.

4 Calibration

I calibrate the model to match salient characteristics of the Austrian labor market. The parameter values are summarized in Table 2. The model is calibrated at the quarterly frequency.

I need to choose values for the time preference \( r \), the vector of the firm-level productivity values \( \mathbf{A} \) and the associated intensity matrix \( \mathbf{\Omega} \), the learning parameters \( \sigma_x, \mu_H, \mu_L \) together with the distribution for \( p_0 \), the flow value of unemployment \( b \), the relative arrival rate of the offers for employed and unemployed \( \psi \), mass of firms \( K \), and the parameters in the vacancy posting cost \( c_0, c_1 \) and in the matching function \( M_0, \alpha \).
<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_u$</td>
<td>vacancy meeting rate for non-employed</td>
<td>0.45</td>
</tr>
<tr>
<td>$b$</td>
<td>value of leisure</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>output noise</td>
<td>9.33</td>
</tr>
<tr>
<td>$\mu_H = -\mu_L$</td>
<td>high match quality</td>
<td>3.5</td>
</tr>
<tr>
<td>$\psi$</td>
<td>relative search intensity of employed and unemployed</td>
<td>0.35</td>
</tr>
<tr>
<td>$\bar{p}_0$</td>
<td>mean of initial belief distribution</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_{p0}$</td>
<td>st. dev. of initial belief distribution</td>
<td>0.1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$\mu$-switching probability (technology adoption shock)</td>
<td>0.25</td>
</tr>
<tr>
<td>$c_1$</td>
<td>curvature of vacancy costs</td>
<td>0.3</td>
</tr>
<tr>
<td>$\delta$</td>
<td>exogenous separation rate</td>
<td>0.0225</td>
</tr>
<tr>
<td>$\Omega = {\omega_{ij}}_{i,j=1}^{N}$</td>
<td>transition matrix for $A$</td>
<td>$\rho = 0.96$</td>
</tr>
</tbody>
</table>

**Assigned parameters**

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>time preference</td>
<td>0.012</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>matching function elasticity</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 2: Parameters of the model and their values. Parameterization is at the quarterly frequency.

To solve for the steady state equilibrium, it is sufficient to choose the endogenous job finding rate $f_u$ and leave the matching function parameters $M_0$, the fixed costs of vacancy posting $c_0$ and the number of firms $K$ unspecified. This is possible because these parameters do not directly enter the value function for the surplus, and, as shown in Appendix B.1, equation (16), they do not affect the distribution of vacancies, $\{\bar{v}_i\}_{i=1}^I$. For given values of $\alpha$ and $K$, it is always possible to find the values of $c_0, M_0$ such that the chosen $f_u$ is consistent with the equilibrium. Moreover, one of the parameters $c_0, M_0$ can be normalized. Appendix B.3 shows how to recover these parameters once we solve for the equilibrium.

To guide the calibration, I split the parameters into groups based on whether they primarily affect the job flows, the worker flows or the shape of the hazard rate of separation. I then use relevant moments from the data to set their values.

I start with the firm-level productivity parameters. I choose the Markov chain for the productivity process $\hat{A}$ to approximate an estimated AR(1) process for the dynamics of the firm-level employment. In particular, I estimate the equation

$$\log EMP_{e,t} = const + \rho \log EMP_{e,t-1} + \sigma_\varepsilon \varepsilon_{e,t},$$

where $EMP_{e,t}$ is employment in firm $e$ at time $t$, $|\rho| < 1$ is the autocorrelation coefficient,
Table 3: Empirical moments targeted in the calibration.

<table>
<thead>
<tr>
<th>Empirical moment</th>
<th>Target</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>unemployment rate</td>
<td>12.0%</td>
<td>13.2%</td>
</tr>
<tr>
<td>job-finding rate for non-employed</td>
<td>43.8%</td>
<td>45.0%</td>
</tr>
<tr>
<td>HR of separation at $\tau = 2$</td>
<td>21.0%</td>
<td>22.2%</td>
</tr>
<tr>
<td>HR of separation at $\tau = 8$</td>
<td>6.0%</td>
<td>6.1%</td>
</tr>
<tr>
<td>HR of separation at $\tau = 10$</td>
<td>3.3%</td>
<td>4.5%</td>
</tr>
<tr>
<td>HR of separation at $\tau = 40$ in non-growing firms</td>
<td>2.3%</td>
<td>2.3%</td>
</tr>
<tr>
<td>shift of the HR in growing firms</td>
<td>1%</td>
<td>1%</td>
</tr>
<tr>
<td>share of separations to other job</td>
<td>0.33</td>
<td>0.31</td>
</tr>
<tr>
<td>hiring rate, as share of employment</td>
<td>9.5%</td>
<td>9.9%</td>
</tr>
<tr>
<td>job creation rate</td>
<td>4.4%</td>
<td>4.3%</td>
</tr>
<tr>
<td>autocorrelation of log-employment</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>standard deviation of log-employment</td>
<td>0.35</td>
<td>0.35</td>
</tr>
</tbody>
</table>

$\epsilon_{e,t}$ is an iid error and $\sigma^2_{\epsilon}$ is the variance of the innovations. I find that the quarterly autocorrelation is $\rho = 0.96$ and the standard deviation is $\sigma_{\epsilon} = 0.32$. I employ the Rouwenhorst method as described in Kopecky and Suen (2010) to approximate this AR(1) process by a 5-state Markov process. Since the model is invariant to additive shifts in the pair $(A, b)$, I normalize the mean value of $A$ to zero.

The dispersion in the match qualities $\mu_L, \mu_H$ relative to the dispersion of the productivities in the vector $A$ determines the relative importance of the worker-level and firm-level heterogeneity. The dispersion in $A$ primarily determines the difference in hiring rates across firms. On the other hand, the dispersion in $\mu$ influences the within-firm heterogeneity in perceived match qualities across tenures, as the uncertainty about the match quality is resolved over time during an employment spell. Similarly as before, the location of $\mu$ is not important: adding a constant to $\mu_H, \mu_L$ and $b$ will have no impact on the equilibrium. I thus normalize $\mu_H = -\mu_L$.

The signal-noise ratio $(\mu_H - \mu_L)/\sigma_x$ and the distribution of the initial match quality determine the slope of the hazard rate of separation shown in Figure 3. I choose initial beliefs to be normally distributed with mean $\bar{p}_0$ and standard deviation $\sigma_{p_0}$, truncated to the interval $[0, 1]$. I choose the parameters $\bar{p}_0, \sigma_{p_0}$ together with $(\mu_H - \mu_L)/\sigma_x$ to match the aggregate hazard rate of separation at different tenures. In particular, I aim to match the hazard rates at 2, 8, 10 quarters of tenure.

The hazard rate at very long tenures is mainly driven by the exogenous separation intensity $\delta$ and the technology adoption shock $\gamma$. To understand this mechanism better, suppose
that both these parameters are zero. In this case, the workers who become confident enough that their match quality is high will never separate from a firm. Since, on average, the confidence about the high match quality is increasing in tenure, this implies that the long-tenure workers have a negligible hazard rate of separation.

Since exogenous separations hit all workers equally, the value of $\delta$ will constitute a lower bound on the separation rate. Empirically (see Figures 4 and 5), the lowest hazard rate of separation exists for workers at long tenures in firms that do not grow or shrink. I therefore choose the quarterly value of $\delta$ to be 2.25%.

On the other hand, the technology adoption shocks occur only when a firm experiences changes in its productivity level which induce a firm to grow or shrink. To isolate the effect of the technology adoption shocks, I look at the shift in the hazard rate of separation at high tenures for workers in firms which grow between 5–20% in a given quarter relative to those in firms which do not grow. This difference determines the probability $\gamma$ that a worker loses her high skill after a firm-level shock.

The decision of the worker to separate to unemployment depends mainly on the consumption flow $b$ and the rate at which a worker meets a vacancy while unemployed, $f_u$. I set the parameter $b$ to match the non-employment rate, which is 12% during the analyzed period. The non-employment rate includes unemployed workers and workers who are out of labor force but become employed again within 5 years. The out of labor force status is not directly observable in the dataset, but I define a worker to be out of labor force if she disappears from the dataset but reappears within 5 years.

I measure the job-finding rate for non-employed workers directly in the data. In particular, I count how many people from the pool of non-employed workers find a job within a given month. The average monthly job-finding rate in period 1986–2007 is 14.6% which implies a quarterly job-finding rate of 0.438. To match this value, I choose the vacancy meeting rate for the non-employed $f_u = 0.45$. Since an unemployed worker does not always accept an offer (she might draw a very low belief), the vacancy meeting rate is higher than the measured job-finding rate.

Employed workers are contacted by other firms at a different rate than unemployed workers. To set the relative intensity of the contact rate, $\psi$, I use data on job-to-job transitions. Out of all workers who separate, one third starts a new job within one month of the termination of the previous employment spell, which is the share targeted in the calibration as direct job-to-job transitions.

Finally, I set the discount rate to $r = 0.0125$ which corresponds to a 5% annual interest
rate. I follow the literature and impose that the elasticity in the matching function is \( \alpha = 0.5 \).

5 Labor market implications

I use the calibrated model to study the joint cross-sectional distribution of firms’ growth rates and workers’ hazard rates of separations across tenure, and to evaluate the importance of the different separation channels for explaining the magnitude of the worker flows. I argue that the model delivers empirically relevant results and replicates the cross-sectional patterns observed in the data.

I solve the model numerically and simulate it. The details of the numerical procedure and simulations are described in Appendix B.1 and B.2. I time-aggregate the simulated paths to construct quarterly time series for employment, hires, separations and tenure distribution for each firm, and repeat the analysis I conducted with the empirical data.

5.1 Cross-sectional distribution of worker and job flows

I start by examining the predictions from the model for the cross-sectional distribution of the worker and job flows. I use the simulated quarterly time series for the hiring, separation and employment growth rates. I sort firms into growth bins and calculate the employment-weighted hiring and separation rates for each bin. Figure 7 depicts the results. The pattern is consistent with the empirical job and worker flow dynamics. There is simultaneous hiring and separations across the whole range of the growth rates. The separation rate is increasing with the contraction rate in the shrinking establishments, and it is growing with the growth rate in the growing establishments.

To understand the mechanism which generates the observed pattern between the firm growth rate and worker separations, consider a firm which has been in its steady state level of employment for some time and receives a better productivity shock. Upon arrival of the shock, the separation threshold \( p \) shifts down. This has an immediate impact on workers close to the separation threshold who instead of separating to unemployment are now willing to tolerate a poor quality match. Through this channel the separation rate of the short-tenure workers decreases since, on average, workers with a low \( p \) are those with short tenure.

The technology adoption shocks affect workers with any belief \( p \). The high-quality workers in a firm face a positive probability that they will not be able to adopt the newly implemented technology, and thus become less valuable to the firm. Facing this possibility, workers update their beliefs downward which moves them closer to the separation threshold.
Figure 7: Comparison of empirical and model-implied hiring and separation rates as a function of the firm growth rate. The figure shows the hiring and separation rates at firms with different growth rates, controlling for the establishment fixed effects. The solid lines represent the model simulations, the dotted lines depict the data, firm growth rates are quarterly.

Workers whose match quality is switched to low will then on average experience a sequence of bad output realizations that will generate a downward drift in the realized belief path, until they eventually reach the separation threshold and leave to unemployment. This technology adoption channel can have a significant effect on the separation rate. Due to selection, firms have a higher share of workers who are good matches, in particular among those with high tenures. The technology adoption shock effectively generates new low-quality matches across all tenures in times when the firm experiences a positive technology shock, thus increasing the separation rate in growing firms.

The mechanism is similar in firms which experienced an adverse productivity shock. The separation threshold shifts up, inducing vulnerable workers with a low belief $p$ to separate immediately. This increases the separation rate of low-tenured workers. At the same time, the technology adoption shock switches some matches from high to low quality, which subsequently further increases the separation rate, as the new low-quality matches drift toward the separation threshold.

The model predicts that the separation rate is increasing in the growth rate in the growing firms, due to two effects. First, growing firms have a higher share of short-tenure workers who face the highest risk of being separated. Second, due to the technology adoption shocks, an increase in the productivity level is associated with a higher risk of being separated even for workers with longer tenures. The fact that the hazard rate of separation in firms with the positive growth is higher than in firms with zero growth across all tenures indicates that
both of these mechanisms play a role.

The relationship between the hiring rate and the firm growth rate generated by the model is consistent with the data. The model predicts that the hiring rate is positive for the shrinking establishments, and positive and increasing with the growth rate in the growing establishments. The hiring policy in the model is such that the number of newly hired workers depends only on the current productivity level of the firm. Thus, as long as the value of a new worker is positive, a firm keeps posting vacancies even if its own productivity is such that incumbent workers tend to leave to unemployment or other jobs faster. Thus, even shrinking firms can have a positive hiring rate.

Finally, Figure 8 shows the distribution of the employment-weighted growth rates generated by the model and as observed in the data. The model does well along this dimension even though it somewhat underpredicts the share of workers in the non-growing firms.

5.2 Hazard rate of separation at different tenures

The key prediction of the learning dynamics is a decreasing relationship between the hazard rate of separation and worker’s tenure. This is due to selection: workers who learn that their match quality is low separate, and thus long-tenure workforce consists primarily of the high-quality matches, with a small hazard rate of separation. As a result, the probability that a match ends decreases with tenure, even though it can be increasing in the early phases.

Figure 9 shows the hazard rate of separation in firms with different growth rates as predicted by the model. This graph is directly comparable to the empirical results shown in
Figure 9: Model-implied hazard rate of separation at different tenures, in firms with different growth rate. Source: model simulations.

Figure 4. The hazard rates are increasing in very early tenures in growing firms and soon start to decline. The model is not able to capture the very high separation rate for tenures shorter than one year in growing firms but the pattern for tenures beyond one year is very similar to data.

Compared to the firms which do not grow, the hazard rate at all tenures is higher in firms with positive employment changes (top panel of Figure 9). This is the consequence of the technology adoption shock mechanism described above. In shrinking firms, the hazard rate of separation across tenures increases with the contraction rate. The differences in separation rates between shrinking and non-shrinking firms are much larger than for growing firms, both in the data and in the model. An adverse productivity shock induces some workers with lower tenures to leave, while the technology adoption shock switches some high-quality matches to low quality. Here, both forces work in the same direction, while they counteract
Table 4: Decomposition of the separation rate according to its source. All numbers are shown as a share of employment. The first line shows the decomposition of the separation rate using the threshold movements as described above. The remaining lines correspond to a version of the model where I shut down the individual channels, namely the firm-level productivity shocks and search on the job.

5.3 Decomposition of the worker flows

Apart from explaining the joint dynamics of firms’ growth rates and workers’ separation rates, I also focus on the decomposition of workers’ separations. I use the model to decompose the separations according to three fundamental causes captured by the model — worker-level shocks, firm-level shocks, and outside offers from other employers.

I carry out the decomposition in two ways. First, I count workers in the simulated data who separate due to hitting the separation threshold, due to the threshold shift, or because they accept an outside offer. Second, I turn off individual separation channels and examine the magnitude of worker flows in these modified models.

The first row in Table 4 shows the results for the full model. In the calibrated model, the quarterly separation rate is 9.9%, somewhat higher than in the data (see Table 3). The share of workers who separate to unemployment is 6.9%, while 3.0% of the workforce separates directly to another job. Decomposing the separations to unemployment further, 3.0% of labor force separates because the firm receives a new productivity shock, while the remaining 3.9% of workers join the unemployment pool because their perceived match quality becomes too low.
The worker-level shocks thus account for 40% of all separations, constituting the most important driving force of the worker flows. Around 30% of separations can be attributed to on-the-job search. Only about 30% of all separations are due to firm-level shocks.

All mechanisms in the model are connected through the belief about the match quality. In particular, compared to a worker with a high belief $p$, a worker with a lower belief $p$ is not only more likely to hit the separation threshold within a given time period, but is also more likely to accept an outside offer or be affected by an adverse productivity shock. Thus, one potential concern with the results is that the decomposition mechanically attributes a large share of separations to the worker-level shocks. To address this issue, I carry out the decomposition in an alternative way. I solve and simulate modified models where I shut down individual sources of workers’ separations and compare the worker turnover in these models with the benchmark.

To shut down the productivity shocks at the firm level, I set all values of the vector of productivities $A_i$ to its mean value, $A_i = 0, \forall i$. Since the change in the match quality is tied to the arrivals of new productivity shocks, I keep the transition dynamics for the Markov chain for $A$ unchanged, with the intensity matrix $\Omega$. This ensures that the technology adoption shocks arrive at the same rate as in the baseline calibration. To turn off the on-the-job search channel, I set the contact rate to zero for the employed worker, $\psi = 0$.

The results are reported in Table 4, rows 2–4. The second row corresponds to the model with no firm-specific shocks. The model still generates a quarterly separation rate of 7.8% of total workforce, indicating that the firm-level shock play an even smaller role than implied by the decomposition in the full model — the separation rate declined only by about 21% relative to the result from the full model. Worker-level shocks now account for three quarters of all separations, while worker turnover due to on-the-job search declines relative to the benchmark. The reason is that without firm-level heterogeneity, there is no mechanism that channels workers from low-productivity firms to high-productivity firms through direct job offers while on the job.

The third row shows results for a model without on-the-job search. In this case, total worker turnover decreases to 7.8% of workforce every quarter but both remaining sources of worker separation increase in magnitude. In the full model, workers who are perceived to be of low match quality or those in low-productivity firms may receive a better job offer and leave immediately to another job. If outside job offers do not arrive, they will wait until their current job position worsens sufficiently to justify the separation to unemployment. This increases the number of transitions through unemployment, despite the decreased total
Finally, I consider the case without firm-level shocks and on-the-job search. In this model the total quarterly worker turnover reduces to 6.7% of workforce. This is still almost three quarters of the turnover in the full model, indicating that worker-level match quality dynamics is the primary source of worker flows.

6 Effects of dismissal costs

In this section, I analyze the effects of the introduction of dismissal costs on the labor market equilibrium. In the model, labor market policies impact job flows both through the hiring as well as through the separation margin. The two-sided heterogeneity introduced in this paper influences both margins and plays a crucial quantitative role in the analysis because the policy have a differential impact on firms with different productivity levels, and thus influence the cross-sectional equilibrium distribution of workers across these firms. I compare these policy experiments with the results in the models of Hopenhayn and Rogerson (1993) and Pries and Rogerson (2005), where the two-sided heterogeneity is absent and changes in the cross-sectional allocation of workers into firms are immaterial.

The dismissal costs are modeled as a fee that the firm has to pay to the government for every employment reduction. I assume that only continuing matches are subject to dismissal costs. Thus, if a worker and a firm meet and decide not to form a match based on their prior belief, no costs are incurred. Moreover, the firm does not pay the dismissal cost if a worker quits to accept an offer from another firm.

Consider first an existing match and, as before, denote its surplus $S(A_i, p)$. The value function is given by

$$rS(A_i, p) = y(A_i, p) - b + \frac{1}{2} \sigma^2_p(p) \frac{\partial^2 S(A_i, p)}{\partial p^2} - \delta (S(A_i, p) + d)$$

$$+ \sum_{j \neq i} \omega_{ij} (S(A_j, p') - S(A_i, p)) \quad \text{if} \quad (A_i, p) \in C$$

$$S(A_i, p) = -d \quad \text{if} \quad (A_i, p) \in C^C$$

where $C$ denotes the region where the match is preserved. The dismissal cost only appears in one place in the value function — an existing match will dissolve if its surplus falls below the cost $-d$. Since the firm does not pay the dismissal cost if a worker quits to another job, the expected surplus gain from searching on the job is zero and this term does not appear in the
value function, similar to the case with no dismissal costs. Even though the surplus of the existing match could be negative, a new match will only be formed if $S(A_i, p) > 0$. Thus, the decision to separate from a match will be characterized by a different set of thresholds than the decision to form a match, the latter being higher than the former.

The obligation to pay the dismissal costs at separation lowers the separation threshold for existing matches as the firm is now more willing to tolerate a bad quality match. The firm wants to postpone separation due to discounting of the separation cost paid at a later date, in hope that the match quality may recover, or that the worker finds a better employment opportunity while searching on the job, in which case the firm avoids paying the dismissal cost. This decision to postpone separations tends to lower the unemployment rate but at the same time decreases the average productivity of existing workers because firms retain a larger share of low quality matches.

At the same time, the dismissal costs have an impact on the hiring thresholds. These thresholds shift up, and thus a better signal $p_0$ is required for a worker and a firm to agree to create an initial match. Firms take into account the present discounted value of firing costs and are willing to hire only more promising workers. The large shift in the hiring thresholds induced by this mechanism has a significant effect on the vacancy posting decision. Firms anticipate that once they meet a worker, it will be more difficult to form a match because the initial belief about its quality may not be sufficiently high. The expected value from meeting a worker decreases, and firms post fewer vacancies.

While this mechanism is active even in models with homogeneous firms and/or workers, the interaction of firm- and worker-level heterogeneity generates an important cross-sectional redistribution in the vacancy posting decisions. The impact of dismissal costs is stronger for firms with lower productivities — they understand that the workers they hire will tend to separate faster, as the separation threshold for a firm with a lower productivity lies higher. This implies that in equilibrium, a larger share of vacancies will be posted by high-productivity firms, and these firms are more likely to tolerate matches even if the belief about match quality deteriorates. This worker reallocation effect pushes down average hazard rates of separation, and can in principle lead to a lower unemployment rate under the dismissal cost policy.

As in Hopenhayn and Rogerson (1993) and Pries and Rogerson (2005), I calibrate the dismissal to be equal to the average annual wage in the economy. Because the equilibrium in the model is invariant to shifts in the level of unemployment benefits and productivities, I need to set the level of these variables appropriately.
Pries and Rogerson (2005) set the dispersion in wages in a firm to be 25%. I mimic this strategy, and pin down the ratio for the value of the match of the most and least productive worker in the firm:

\[
1.25 = \frac{J(w, A_i, 1) + W(w, A_i, 1)}{J(w, A_i, g(A_i)) + W(w, A_i, g(A_i))} = \frac{S(A_i, 1) + B}{S(A_i, g(A_i)) + B}
\]

The last expression depends on match surplus \( S(A_i, 1) \) which is determined uniquely regardless of the shifts in the value of unemployment \( B \), and thus allows me to pin down \( B \) (the surplus of the match at the separation threshold is zero). Because firms are heterogeneous, I choose to determine \( B \) using the firm with the median productivity in the distribution of firms weighted by their size.

With \( B \) in hand, I can compute the average quarterly flow value of an existing match as

\[
3 \times [\bar{S} + B] (r + \delta)
\]

where \( \bar{S} \) is the average surplus of an existing match

\[
\bar{S} = \sum_i \int S(A_i, p) \frac{g(A_i, p)}{E} dp
\]

and \( r \) and \( \delta \) are the monthly discount rate and exogenous separation rate, respectively.

Table 5 summarizes the results of the policy implementation under alternative counterfactual scenarios. The first two columns compare the results from the benchmark calibration to those generated under the dismissal cost policy.

The introduction of dismissal costs generates an increase in the unemployment rate of about 1.4 percentage points, and increase the duration of unemployment by about 10%. Average productivity of employed workers also increases, by about 0.8%. These results
are driven by the two crucial forces in the model — shifts in the hiring and separation thresholds. These shifts will lead to a change in the distribution of vacancies across firms, and subsequently to shifts in the stationary distribution of workers with different match qualities.

Figure 10 documents the forces underlying the mild increase in productivity. Since the separation thresholds shift down, firms are more willing to tolerate low quality matches. This will tend to shift the distribution of workers toward worse match qualities, and the figure shows that this effect is particularly pronounced for the high productivity firms. At the same time, the shift in the hiring thresholds will be more pronounced for the low productivity firms. These firms face a higher likelihood that they will separate the worker quickly, and thus the present discounted value of the dismissal costs is higher for them. This effect will imply that in equilibrium, the high productivity firms will post a larger share of all vacancies and also attract a larger share of new workers.

In principle, the total equilibrium effect of the dismissal cost policy on the unemployment rate and the average productivity is ambiguous. The unemployment rate will be pushed up through the overall decrease of the willingness of the firms to hire new workers but at the same time pushed down by the downward shift in the separation thresholds as well as by the shift of the new hires toward more productive firms which are more likely to retain workers with lower match qualities. The average productivity will be co-determined by the same factors. Higher hiring thresholds will tend to increase average productivity, as only workers with better signals are hired, while lower separation thresholds will counteract this effect. The tilting of the distribution of vacancies toward high productivity firms will also have a positive effect on average productivity.

Pries and Rogerson (2005) find that the introduction of a dismissal cost that corresponds to a 3-month wage increases the unemployment rate by 1 percentage point and increases the productivity by about 1 percentage point. In my model (second column of Table 5 — benchmark with dismissal costs), I find similar effects but the underlying mechanism is to a large extent determined by the shift in the distribution of vacancies toward more productive firms. As a counterfactual experiment, I introduced the dismissal cost while keeping the distribution of vacancies (but not their overall level) fixed at the benchmark values. In this version of the experiment, the unemployment rate increased by 1.6 percentage points and the average productivity increased only negligibly, by 0.06%. The equilibrium changes in the distribution of vacancies in response to the dismissal cost policy, absent in Pries and Rogerson (2005), thus reduce the unemployment rate modestly but have a large effect on
average productivity of employed workers.

In order to better understand the role and interaction of different driving forces underlying the labor market flows, I construct counterfactual economies in which I shut down individual transmission channels in the model, and study the impact of the dismissal cost policy in these economies. In order to achieve comparability, I recalibrate the exogenous separation rate and the relative search intensity of employed and unemployed workers to fit the unemployment rate and the unemployment duration of the benchmark model (first column of Table 5). The last three columns of Table 5 show the results for the versions of the model without learning (fixed $p$), without on the job search ($\psi = 0$) and without the technology adoption shocks ($\gamma = 0$).

Consider first the model without learning. In this version of the model, the worker-firm pair forms a belief about the quality of their match when they meet, but never update this belief again, and the actual productivity is equal to their belief. I also switch off the technology adoption shocks, so that the belief does not change even if the firms receives a productivity shock. Workers therefore separate only exogenously or if the firm receives an adverse productivity shock that moves the separation threshold above the match-quality
belief.

The dismissal costs lead to an increase in the unemployment rate that is almost identical to that in the benchmark model but the effect on the unemployment duration is much more pronounced. With fixed beliefs, dismissal costs generate a larger gap between the hiring and separation threshold than in a model with learning, and therefore, both hiring and separations drop more. The effects on unemployment offset each other but the duration of unemployment increases more.

The magnitude of the effect of the dismissal cost policy on the unemployment rate and average productivity depends on the ability of workers to search on the job. When employed workers are allowed to search, incentives of firms to dissolve the match and pay the dismissal costs are low because the firm is willing to postpone separation of a low quality match in the hope that the worker switches to another job directly. Thus, the option of search on the job mitigates the negative effect of dismissal costs. The second-last column of Table 5 shows that when search on the job is not allowed, the unemployment rate increases by more than 3 percentage points. Without search on the job, firms will not postpone separations of unproductive workers due to the search option, and consequently impose higher standards at hiring. Both these effects will increase the unemployment rate. At the same time, since firms only retain more productive workers, average productivity of employed workers increases markedly.

This experiment highlights two important takeaways. First, in order to pin down the right magnitude of firms’ incentives to postpone separation, it is crucial that the model generates the correct relative shares of job-to-job and job-to-unemployment transitions. Second, there is a notable tradeoff between unemployment and average productivity of employed workers. This sorting and selection mechanism indicates that unemployment may rise without a significant decline in total output.

Finally, suppose that one fails to include technology adoption shocks into the model, and instead assigns a higher intensity to exogenous separations in order to keep the unemployment rate unchanged. The key difference between models with and without technology adoption shocks is the timing of separations. The technology adoption shock does not lead to an immediate separation. Rather, if the match becomes of low quality, the worker-firm pair still needs some time to learn about it and the separation occurs once the belief hits the separation threshold.

At the same time, declines in match quality in a model with technology adoption shocks are correlated with productivity shocks, which decreases the option value of postponing a
separation in a low productivity firm. Such a firm is willing to postpone separation and retain a low quality worker in the hope that the firm-level productivity increases. If the firm understands that the arrival of the high productivity shock is associated with an expected decline in match quality, the motive to postpone separation is diminished.

Therefore, in a model without technology adoption shocks, the separation thresholds will be lower, and in order to preserve the aggregate unemployment rate, the exogenous separation rate must be higher. The introduction of the dismissal cost policy then leads to a smaller shift in the separation thresholds. The impact of separations is therefore larger — the unemployment rate and duration of unemployment increase more than in the benchmark model. The average productivity of employed workers also increases more, due to a lower willingness of firms to retain low quality workers.

7 Conclusion

I study the relative importance of different forces that contribute to the large observed magnitude of labor market flows. I distinguish idiosyncratic firm-level productivity shocks, worker-level shocks captured by the learning about match quality, and job switches due to on-the-job search. I develop a theoretical model that includes these forces, and assess their relative importance using data from Austrian social security records.

From a theoretical perspective, an important new element of the model is to allow for a correlation between technology shocks at the firm level and the firm’s match qualities with individual workers. A positive productivity shock does not automatically make all workers more productive. Some workers might not be able to adopt the new technology, and even though they used to be well-suited for the job when the old technology was in place, with the arrival of the new technology the quality of the match decreases. I model this mechanism as a switch in the quality of the match that occurs randomly when a new firm-level productivity shock arrives. Since the quality is unobserved, the switch is unobserved as well, and the worker-firm pair has to learn about it again. Absent the technology-adoption shocks, the long-tenured workforce would consist primarily of the high-quality matches with negligible hazard rates of separation. This would be at odds with the data where even long-tenured workers face a non-negligible risk of separating from their current job that is higher in growing firms than in firms that do not grow.

The results show that worker-level shocks are the dominant driving force of worker flows. In the full model, separations to unemployment due to worker-level shocks account for 40%
of all separations, while an additional 30% are due to direct job-to-job transitions. About 30% of all separations are attributable to firm-level productivity shocks.

The importance of firm-level shocks for overall worker flows is even more limited when studying counterfactual models. In the counterfactual world without any firm-level shocks, worker flows decline only by about 21% relative to the benchmark model. Both the results from the full model as well as from the counterfactual exercises indicate that worker-level heterogeneity is crucial for understanding the dynamics of labor market turnover.

Finally, I examine effects of dismissal costs on equilibrium unemployment rate, unemployment duration and the average productivity and show that the magnitudes depend on which labor flows forces operate in the model. The match-level heterogeneity captured by the distribution of beliefs and the firm-level productivity shocks play a crucial role. Dismissal costs shift the distribution of vacancies toward more productive firms which are those more tolerant of the low-quality matches. Thus, depending on relative magnitudes of the match-level and firm-level heterogeneity, dismissal cost could have in principle positive or negative effect on average productivity. I find that dismissal costs increase unemployment rate rather modestly but have a large negative effect on unemployment rate and unemployment duration.

References


Appendix

A Omitted derivations

A.1 Properties of the separation thresholds

1. Assume that $S(A,p)$ is increasing in $A$ and $p$. Then the separation threshold $\bar{p}(A_i)$ is decreasing in $A_i$. For $A_j > A_i$ we have

$$S(A_j, \bar{p}(A_i)) > S(A_i, \bar{p}(A_i)) = 0 \implies S(A_j, \bar{p}(A_i)) > 0 = S(A_j, p(A_j)) \implies p(A_i) > p(A_j).$$

2. The separation threshold $\bar{p}(A_i, A_j, p)$ is decreasing in $A_i$. Consider $A_k > A_i$. Then

$$S(A_j, p_0) = S(A_i, \bar{p}(A_i, A_j, p_0)) < S(A_k, \bar{p}(A_i, A_j, p_0))$$

$$S(A_j, p_0) = S(A_k, \bar{p}(A_k, A_j, p_0)) < S(A_k, \bar{p}(A_i, A_j, p_0)) \implies \bar{p}(A_k, A_j, p_0) < \bar{p}(A_i, A_j, p_0)$$

and therefore it is decreasing in $A_i$.

3. Assume that $S(A,p)$ is increasing in $A$ and $p$. Then the separation threshold $\bar{p}(A_i, A_j, p)$ is increasing in $A_j$. Consider $A_l > A_j$. Then

$$S(A_i, \bar{p}(A_i, A_j, p_0)) = S(A_j, p_0) < S(A_l, p_0) = S(A_i, \bar{p}(A_i, A_l, p_0))$$

$$\implies \bar{p}(A_i, A_j, p_0) < \bar{p}(A_i, A_l, p_0)$$

and thus it is increasing in $A_j$.

4. Assume that $S(A,p)$ is increasing in $A$ and $p$. Then the separation threshold $\bar{p}(A_i, A_j, p)$ is increasing in $p_0$. Consider $p_1 > p_0$. Then

$$S(A_i, \bar{p}(A_i, A_j, p_0)) = S(A_j, p_0) < S(A_j, p_1) = S(A_i, \bar{p}(A_i, A_j, p_1))$$

$$\implies \bar{p}(A_i, A_j, p_0) < \bar{p}(A_i, A_j, p_1)$$

and thus it is increasing in $A_j$. 

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A.2 Distribution of \((A, p)\) in the model with dismissal costs

In the model with the dismissal costs, the Kolmogorov forward equation has the following form,

\[
0 = \frac{d^2}{dp^2} \left( \frac{1}{2} \sigma_p^2 (p) g(A, p) \right) + \sum_{j \neq i} \omega_{ji} \frac{g(A_j, \frac{p}{1-\gamma})}{(1-\gamma)} + f_e \bar{v}_i g_1(p) I_{S(A_i, p) > 0} \left[ \sum_j \int_{0}^{1} g(A_j, \bar{p}) \, d\bar{p} \right] + f_u \bar{v}_i g_0(p) I_{S(A_i, p) > 0} \left[ 1 - \sum_j \int_{0}^{1} g(A_j, \bar{p}) \, d\bar{p} \right] - f_e \sum_{j=1}^{I} \bar{v}_j \int_{p:S(A_j, p_0) > S(A_i, \bar{p})}^1 g_0(p_0) \, dp_0 \right] g(A_i, p) - \sum_{j \neq i} \omega_{ij} g(A_i, p) - \delta g(A_i, p),
\]

where \(I_x\) is an indicator function for event \(x\). It features the same channels as Kolmogorov equation in the model without the costs, but here the flows between jobs and flows from unemployment to employment have to be conditioned on the match having a positive surplus.

The expected value from randomly meeting a worker is

\[
\bar{J}(A_i) = \frac{U}{L} \int S(A_i, p_0) \, dp_0 + \psi \frac{E}{L} \sum_{j=1}^{I} \int \int I_{S(A_i, p_0) > 0} \left( S(A_i, p_0) - S(A_j, p) \right) + \frac{g(A_j, p)}{E} g_0(p_0) \, dp_0 \, dp_0.
\]

B Numerical procedure and simulations

B.1 Numerical procedure

I solve the system of ODEs \((8)\) numerically. I create a grid for \(p\), call it \(\{p_k\}_{k=1}^{K}\) and discretize the system.

For known values of the separation thresholds \(\{p(A_i)\}_{i=1}^{I}\), the discretized equations constitute a system of linear equations in unknowns \(\{S(A_i, p_k)\}_{i=1, ..., I, k=1, ..., K}\). However, the values of \(\{p(A_i)\}_{i=1}^{I}\) are determined endogenously within the system and thus I need to iterate:

Step 1: Guess values for \(p(A_i), \forall i = 1, \ldots, I\).

Step 2: Solve the discretized system for the guessed values of \(\{p(A_i)\}_{i,j=1}^{I}\).
Step 3: Verify the accuracy of the guess for $p(A_i)$ by checking whether the smooth-pasting conditions $\partial g(A_i, p(A_i)) / \partial p = 0$ are satisfied. Update the guess for $p(A_i)$ and repeat the step 2 until the precision criterium for $p(A_i)$ is met.

After solving for the surpluses $S(A_i, p)$, I calculate the rest of the model in the following way.

Step 4: Guess values for $\bar{v}_i$, $\forall i = 1, \ldots, I$.

Step 5: Solve the stationary distribution $g(A_i, p)$ using the Kolmogorov forward equation. Use that to calculate the vacancy posting decision of firms.

Step 6: Verify whether the firm’s vacancy posting decision is consistent with the guess for $\{\bar{v}_i\}_{i=1}^N$ by checking whether the condition (16) (derived below) holds:

$$\bar{v}_i = \frac{\bar{\omega}_i \bar{J}(A_i)^{1/c_1}}{\sum_j \bar{\omega}_j \bar{J}(A_j)^{1/c_1}}. \tag{16}$$

Update the guess for $\bar{v}_i$ and repeat the steps 5-6 if not.

The condition for the distribution of vacancies (16) is given by the decision of an individual firm on how many vacancies to post and the share of these firms in equilibrium. A firm of type $A_i$ posts $v(A_i)$ vacancies such that the marginal costs of posting vacancies equals expected benefit from filling a vacancy:

$$C'(v) = c_0 v^{c_1} = q \bar{J}(A_i)$$

$$v(A_i) = \left( \frac{q}{c_0} \bar{J}(A_i) \right)^{1/c_1}.$$

In the stationary equilibrium, the share of firms with productivity $A_i$ is $\bar{\omega}_i$ where $\bar{\omega}_i$ is the $i$-th element of $\bar{\omega}$ that solves $\bar{\omega} \Omega = \mathbf{0}$, normalized by $\bar{\omega} \mathbf{1} = 1$. The share of firms together with the number of vacancies give us the expression for the distribution of vacancies:

$$\bar{v}_i = \frac{\bar{\omega}_i v(A_i)}{\sum_j \bar{\omega}_j v(A_j)} = \frac{\bar{\omega}_i \left( \frac{q}{c_0} \bar{J}(A_i) \right)^{1/c_1}}{\sum_j \bar{\omega}_j \left( \frac{q}{c_0} \bar{J}(A_j) \right)^{1/c_1}} = \frac{\bar{\omega}_i \bar{J}(A_i)^{1/c_1}}{\sum_j \bar{\omega}_j \bar{J}(A_j)^{1/c_1}}.$$

B.2 Simulations

I simulate the model $10^6$ times with one period being one day. I start from the steady state level of employment. Each employed worker draws her value of $\mu$ and is assigned to a firm of type $A$ respecting the stationary distribution $\bar{\omega}_i$. From here, I run the economy forward.

The timing of events within a period is as follows:

1. firms post vacancies, which together with the number of searchers determines $f_u, f_e$ and the distribution of vacancies $\bar{v}_i$,
2. production takes place, workers update beliefs based on the output realization,

3. all workers receive offers according to $f_e, f_u$ and $\bar{v}_i$,

4. workers who met a firm draw an initial belief $p_0$ from the distribution $F_{p_0}(\cdot)$

5. separations take place: i) endogenously if $p < p(A_i)$, ii) via search on the job $S(A_i, p) < S(A_j, p_0)$

6. hiring takes place: employed workers who accepted an outside offer and unemployed workers who got an offer join firms,

7. new matches draw the match quality $\mu$ from distribution $F_\mu(\cdot; p_0)$, respecting their prior belief

8. measurement takes place: I measure hires, separations, employment, and tenure distribution in each firm,

9. in each firm that receives a new productivity shock $A_j$, agents update beliefs to $p'$ and a share $\gamma$ of workers in these firms change the value of the match quality.

I drop first 10000 periods of the data. With the rest, I repeat the analysis I conducted with the empirical data.

B.3 Setting omitted parameters for simulations

The structure of this model allows to find a steady state equilibrium without specifying all deep parameters. These parameters, however, have to be set for the simulations, and in this section I show how to recover them. In particular, I need to find values of $c_0$ and $M_0$ such that the equilibrium is consistent with the job-finding rate $f_u$. In fact, as will be become clear later, one of these values can be normalized.

In the simulations, there are $N$ workers and $K$ firms. The matching market is described by the matching function

$$M = M_0 L^\alpha V^{1-\alpha},$$

where $L = N(u + \psi(1 - u))$ is the number of searchers and $V = K \sum_{i=1}^I \bar{\omega}_i v(A_i)$ is the number of vacancies. Once I solve for the steady state equilibrium, $L$ is known, but the total number of vacancies $V$ is not. Recall that only the distribution $\{\bar{v}_i\}_{i=1}^I$ is needed to find the steady state but the level of vacancies is not. That will be given by the missing parameters $c_0$ and $M_0$.

The form of the matching function implies that

$$f_u = \frac{V}{L}, \quad f_u = M_0 \left(\frac{V}{L}\right)^{1-\alpha}.$$
Using the expression for the distribution of vacancies, \( \bar{v}_i = K \bar{\omega}_i v(A_i)/V \), and the first order condition for the number of vacancies, I express the total number of vacancies as

\[
V = \frac{K \bar{\omega}_i v(A_i)}{\bar{v}_i} = K \frac{\bar{\omega}_i}{\bar{v}_i} \left( \frac{q}{c_0} \bar{J}(A_i) \right)^{1/c_1}.
\]

Next, use that \( V = L f_u/q \) to eliminate \( V \) and rearrange the terms to get:

\[
L \frac{f_u}{q} = K \frac{\bar{\omega}_i}{\bar{v}_i} \left( \frac{q}{c_0} \bar{J}(A_i) \right)^{1/c_1}.
\]

\[
q^{-(c_1+1)/c_1} c_0^{1/c_1} = \frac{1}{L \frac{f_u}{q}} K \frac{\bar{\omega}_i}{\bar{v}_i} \bar{J}(A_i)^{1/c_1}.
\]

The terms on the right-hand side are all known, while \( q \) and \( c \) on the left-hand side still need to be determined. Notice that \( c_0 \) can be normalized. Increasing the constant \( c \) only increases the vacancy-filling rate but does not alter other variables.

I normalize \( c_0 = 1 \). Then the equation above implies the value for \( q \)

\[
q = \left( \frac{1}{L \frac{f_u}{q}} K \frac{\bar{\omega}_i}{\bar{v}_i} \bar{J}(A_i)^{1/c_1} \right)^{-c_1/(1+c_1)},
\]

which in turn is used to find the value of \( B \) from

\[
M_0 = f_u / \left( \frac{f_u}{q} \right)^{1-\alpha}.
\]

### B.4 Numerical procedure for the model with the dismissal costs

There are two issues with the dismissal costs. First, the Kolmogorov forward equation describing the stationary distribution is modified:

Second, I solve the model taking \( f \) as given. However, \( f \) is an endogenous variable, not a parameter. Thus, I need to make it consistent. After I solve for the surplus, which does not depend on \( f \), I proceed then as follows.

Step 1: Guess value \( f \).

Step 2: Guess values for \( \bar{v}_i, \forall i = 1, \ldots, I \).

Step 3: Solve the stationary distribution \( g(A_i, p) \) using the Kolmogorov forward equation and verify the guess for \( \bar{v}_i \) as before.

Step 4: Verify the guess for \( f \). Since I already know the matching parameter \( B \) (recovered as described above), I can use that

\[
q = M_0 \left( \frac{V}{L} \right)^{-\alpha} \Rightarrow V = L \left( \frac{q}{M_0} \right)^{-1/\alpha}
\]
As derived earlier,
\[ V = K \frac{\bar{\omega}_i}{\bar{v}_i} (q \bar{J} (A_i))^{1/c_1} \]
and substituting \( V \) out,
\[
L \left( \frac{q}{M_0} \right)^{-1/\alpha} = K \frac{\bar{\omega}_i}{\bar{v}_i} (q \bar{J} (A_i))^{1/c_1}
q^{-1/\alpha-1/c_1} = \frac{K}{L} \frac{\bar{\omega}_i}{\bar{v}_i} (A_i)^{1/c_1} M_0^{-1/\alpha}
\]
Here all terms on the RHS are known, thus I can calculate \( q \). Then, finally use that
\[
f = q \frac{V}{L} = q \frac{1}{L} S \left( \frac{q}{M_0} \right)^{-1/\alpha} = M_0^{1/\alpha} q^{1-\frac{1}{\alpha}}
\]
to calculate \( f \). If this corresponds to the guess, I stop. Otherwise, I repeat the steps 2-4.

C Data

C.1 Labor market and institutional setting in Austria

The Austrian labor market is characterized by a relatively large turnover of jobs and workers and a low unemployment rate. Pries and Rogerson (2005) use a variety of sources for job and worker flows in the U.S. and Europe to show that even though the magnitude of job turnover is similar in continental Europe and the U.S., worker turnover in the U.S. is higher than in Europe by a factor of at least 1.5. In this respect, worker turnover in Austria is high relative to other countries in continental Europe, as around 9% of workers separate from their employers every quarter and about the same number is hired, compared to 11% in the U.S.

The average unemployment rate in Austria during the 1986-2007 period was 4.5% according to the Labor Force Survey, one of the lowest in Europe. The ASSD is based on administrative data, and thus measures registered unemployment. The unemployment rate according to this measure reaches 6.5% for the same period, which is still lower than unemployment rates observed in large European economies.

A useful feature of the Austrian labor market is the absence of a pronounced business cycle during the period covered by the data. Macroeconomic aggregates and labor market flows have little cyclical variation during this period. There were 2 quarters with a negative rate of GDP growth but this was not reflected in the analyzed labor market indicators.

Worker and job flows are crucially influenced by existing regulations of worker displacement that lead to financial and nonfinancial costs of firing. In this respect, Austria is a country with relatively low firing costs, at least in comparison to continental Europe, and I will abstract from
the firing cost in my model. The analysis of the impact of firing costs and other institutional labor market frictions remains an interesting area of further research.

The main source of the direct financial costs of displacing a worker is the severance payment to the worker. Currently, two systems regulating the severance payment coexist in Austria: an “old” experienced-based system applies to all contracts signed before December 31, 2002, while the “new” contribution-based system covers all contracts signed after this date. In the old system, if a contract is terminated by the employer, a worker with at least 3 years of tenure becomes eligible for the severance payment, starting with a two-month salary and gradually increasing with the tenure of the displaced worker. In the new system, employers do not make any direct severance payments to the displaced worker. Instead, employers contribute a certain percentage of workers’ gross wages into a severance fund. In case of a dismissal, the worker receives a payment from the severance fund.

There is an additional layoff tax for displacing a worker who is older than 50 years and has been continuously employed in the given firm for at least 10 years. The exact amount of the tax depends on the tenure and years to retirement but it can reach 170% of the monthly salary for men and 60% for women.

Job losers are eligible for unemployment benefits if they have worked for at least 12 months during two years preceding the job loss. The benefits on average cover 55% of worker’s previous net wage which provides strong incentives to register. The duration of the benefits depends on the number of months the worker has worked in the past 5 years, and typically varies between 20 and 30 weeks.

C.2 General description of the data

I use the Austrian Social Security Database (ASSD) for the empirical analysis. The ASSD records the spells of individuals that contribute to the determination of eligibility and amount of the social security, sickness, health and accident benefits. These spells are then translated into the labor market status. I distinguish 4 statuses: employed, unemployed, maternity leave and retirement. The spells that are not relevant for the labor market status determination, like for example widower or foster-care benefits, are deleted from the database.

The ASSD is thus a dataset containing labor market histories of almost all individuals in Austria from 1972 to 2007 with observed spells of employment, unemployment, maternity leave

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5 The severance payment rises to 3 months for workers with 5 years of service, to 4 months after 10 years, 6 months after 15 years, 9 months after 20 years and 12 months after 25 years.

6 The contribution is 1.53% of worker’s gross salary. After a dismissal, the worker can leave the benefit in the fund or have it paid out as a lump sum, but the lump sum payment is available only to workers who contributed for at least 3 years into the fund.

7 This law was introduced in 1996. Schnalzenberger and Winter-Ebmer (2009) provide a more detailed description of the law.
and retirement. For each spell I observe its begin and end date. For employed individuals I observe
the establishment code of the employer and annual earnings associated with this employer. If a
worker holds multiple jobs within the same year, I see earnings from each job separately.

The database contains several demographic characteristics, namely the year of birth, gender,
nationality and region of residence. Education is provided by Public Employment Service Austria
(AMS) and therefore is available only for those who went through at least one unemployment spell, which is around 35% of individuals in the sample. The annual income is bottom and top coded. I observe a 4-digit NACE industry code (Statistical Classification of Economic Activities in the European Community) as well as the region of residence for the establishments.

Even though the ASSD covers the vast majority of the Austrian workers since 1972, there are
some groups of workers who used to be exempt from paying the social security contributions, and
thus would not be covered in the database since its beginning. These are government employees
who were added into the dataset only in 1988, free contractual workers who were added in 1996, and
marginal part-time employees working less than 10 hours a week who were added in 1994. I exclude
all three groups from my analysis. I consider only private full-time employees which eliminates the
first two categories. To keep the sample consistent over time, I exclude the free contractual workers
as well, which is only a small share of workers. According to the Austrian Labor Force Survey, the
share of free contractual workers is constant at around 1.5% of total employment.

The dataset underreports unemployment spells for years before 1985 and therefore I focus only

C.3 Construction of the main variables

I use the establishment identification code to group workers into establishments. For each estab-
lishment I construct the quarterly time series of employment, hires and separations using the begin
and end date of workers’ employment spell. I define a worker as employed in establishment $e$ in
quarter 1, 2, 3 or 4 of the given year if she works in establishment $e$ on the reference day March 31,
June 30, September 30 and December 31 of the given year, respectively. In particular, this means
that her spell started at or before and ended at or after the reference day. I consider a worker
to be a hire in quarter $i$ of the given year if the begin date of her employment spell falls between
two reference dates $i$ and $i-1$ of that year. Separations are defined in a similar way: a worker is
counted as a separation in quarter $i$ if the end date of her spell falls between $i$ and $i-1$.

For each establishment $e$, I construct quarterly time series of employed, hired and separated
workers, $E_{et}, H_{et}, S_{et}$. I use them to construct rates for establishment growth $g_{et}$, hiring $h_{et}$ and
separation $s_{et}$ as

$$
g_{et} = \frac{\hat{E}_{et} - E_{et-1}}{\frac{1}{2} (E_{et-1} + \hat{E}_{et})}, \quad h_{et} = \frac{H_{et}}{\frac{1}{2} (E_{et-1} + \hat{E}_{et})}, \quad s_{et} = \frac{S_{et}}{\frac{1}{2} (E_{et-1} + \hat{E}_{et})}
$$
where $\hat{E}_{et} = E_{e,t-1} + H_{et} - S_{et}$ is a revised measure of employment which ensures that hiring, separation and employment growths are consistent,

$$g_{et} = h_{et} - s_{et}. \quad (17)$$

Note that without the revised measure of employment, (17) does not hold. The reason is that workers who separate from their employers exactly on the reference date are counted as separations at time $t$ but also as being employed at time $t$ and thus violate the equation $E_{et} = E_{e,t-1} + H_{et} - S_{et}$.

I define a direct job-to-job transition as a transition where the worker started a new employment spell within 28 days of the termination of the previous one.

C.4 Tenure and hazard rate of separation

I define the tenure of an employee at time $t$ as the difference between the reference date $t$ and the begin date of the worker’s spell. I measure tenure in quarters. Using the measure of tenure, I construct a quarterly time series of tenure distribution for each establishment. Then the quarterly hazard rate of separation at tenure $\tau$ at time $t$ is defined as the share of workers who had tenure $\tau$ at time $t - 1$ and separated during quarter $t$,

$$HR_{\tau,t} = \frac{EMP_{\tau+1,t} - EMP_{\tau,t-1}}{EMP_{\tau,t-1}}. \quad (18)$$

I first pool data from all establishments together and apply the formula above to calculate the aggregate quarterly hazard rate of separation at tenures $\tau = 1, \ldots, 40$ quarters.

To examine how the hazard rates depend on the establishment growth rate, I split the establishments into 6 growth bins, pool the data within each bin together and use (18) to construct the hazard rate curve for each bin. I use the following growth bins: $[-2, 0.2]$, $(0.2, 0.05]$, $(0.05, 0]$,

$[0, 0.05]$, $[0.05, 0.2]$, $[0.2, 2]$. I use an increasing width of the intervals because the number of establishment with a given growth rate declines quickly as one moves away from 0.

C.5 Sample selection

I consider only full-time workers with employment spells of at least 14 days. I merge these workers into establishments as described above and construct quarterly time series for employment, hires and separations. I select establishments which had more than 5 employees in at least one quarter during their lifetime.

This gives me an average sample size of 2.46 million employees and 112 thousand establishments in each quarter.