Inflation, default, and the denomination of sovereign debt

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JOB MARKET PAPER

December 2013

Abstract

Emerging market countries increasingly issue nominal government debt. At the same time, these countries experience sovereign debt crises with default and high inflation. This paper studies the implications of debt denomination for sovereign default and inflation policies. Using bond-level data on government borrowing, I show that default and inflation rates vary systematically with debt denomination: high nominal debt shares are associated with low inflation and default rates. I then build a monetary model of sovereign debt with lack of commitment, in which differences in debt denomination generate this pattern, and the government inflates more when debt is real. Issuing real instead of nominal debt has two effects in the model. On the one hand, real debt reduces the incentive to create costly inflation because the value of the debt is fixed in real terms. It thus helps mitigate the commitment problem. On the other hand, because the commitment problem is smaller, real debt facilitates more debt accumulation over time, causing the government to resort to the printing press after all to finance the debt burden. In a calibrated version of the model this second effect dominates: As in the data, inflation and default rates are higher on average when debt is real instead of nominal. Default risk helps generate large differences in inflation and default rates across debt regimes as the government optimally inflates in order to avoid default.

*Latest version available at `www.econ.umn.edu/~sund0444`. I would like to thank seminar participants at the Minneapolis Fed and members of the Minnesota Trade Workshop, Manuel Amador, Illenin Kondo, Andrea Waddle and David Wiczer for many useful comments, and especially Cristina Arellano, Tim Kehoe and Fabrizio Perri for all the advice and guidance. All errors are mine.

1 Introduction

Emerging market governments actively manage the denomination of their sovereign debt. Brazil has a declared target of 30-35% inflation indexed debt, South Africa aims for 70% nominal debt, citing the composition of debt as "one of the major risk concerns".[1] At the same time, these countries experience debt crises with default and high inflation episodes. The denomination of government debt determines how countries handle such crises: When debt is nominal, the government can reduce its debt burden through inflation or outright default, whereas with real debt, the government can only default on the debt. Theoretical studies on emerging markets debt crises, however, have largely ignored the denomination of debt. This paper studies the effect of debt denomination on inflation, default and debt crises when governments lack commitment to future policies. The main finding is that nominal debt provides incentives for governments to induce paths of lower average inflation, and to default less frequently, because this allows governments to get better terms on their sovereign debt. Issuing real debt does not reduce the government's incentives to reduce inflation sufficiently.

I document that, empirically, emerging market countries borrow largely in nominal terms, that is in terms such that domestic inflation affects the debt burden. I construct measures of government debt stocks based on Bloomberg bond level data for 27 countries that comprise over 80% of all emerging market countries in terms of GDP and all major issuers in emerging sovereign bond markets, including Brazil, Mexico, China and India. I find that on average in this sample over the past two decades, 75% of sovereign bond debt is nominal. Real and foreign currency emerging market government debt is the exception rather than the rule. I then show that inflation and default rates vary systematically with the share of debt that is in nominal terms. Emerging market countries with high shares of nominal debt tend to experience lower than average inflation and default rates. Average inflation in nominal debt countries was around 5%, but as high as 20% annually for real debt issuers. The numbers are similar for default rates.

The paper then builds a dynamic monetary model of sovereign debt with endogenous default that can rationalize this pattern. I consider two environments, one with nominal and one with real debt, to analyze the effects that debt denomination has on inflation and default incentives. On the one hand, denomination affects how strong incentives are

[1] Sources: "Optimal Federal Public Debt Composition: Definition of a Long-Term Benchmark" by the Brazilian Treasury, 2011. "Debt Management Report 2011/2012" by the South African Treasury.

to inflate today. Inflation is more useful when debt is nominal instead of real because it generates seigniorage revenue and erodes the real value of the debt. It is less useful when debt is real since the value of the debt is fixed. On the other hand, the denomination affects borrowing decisions. The government in the model takes into account that more borrowing today increases incentives for future governments to inflate or default. It does so via bond prices, which reflect expected inflation and default. More borrowing will lower bond prices and hence revenues today to the extent that the borrowing creates expectations of future inflation and default. When debt is nominal, borrowing is more restrictive since bond prices fall when inflation expectations rise. When debt is real, bond prices carry no inflation premia so the government can issue bonds without directly reducing revenues through higher inflation premia. It is thus able to borrow more and may inflate after all for sufficiently high debt levels or default risk. The paper finds that in the long run, this second effect dominates and that an economy with real debt has on average higher inflation and default rates.

The paper focuses on an optimal policy problem without commitment. In the model, a benevolent government decides on inflation, default, and new borrowing to finance a stochastic stream of government expenditures and to service the debt. Households provide labor, consume cash and credit goods and lend to the government. Inflation is costly because it distorts cash relative to credit good consumption due to a cash in advance constraint. Default incurs an exogenous resource cost and temporary exclusion from credit markets. As in any Markov environment without commitment, the government takes as given the optimal policy functions of future governments and internalizes the households equilibrium conditions. Importantly, the price of bonds compensates households for future inflation or default risk.

Borrowing in the model is driven by the government's lack of commitment. Bonds provide a lump sum means of raising revenue because bond issuance does not distort private sector allocations. Ex post, however, outstanding debt creates an incentive for the government to inflate and default in order to relax the budget constraint. Incentives to inflate and default are increasing in the level of debt, which is reflected in bond price functions being decreasing in the level of borrowing. Since the government takes into account the effect of borrowing on bond prices, this therefore limits equilibrium borrowing and debt levels.

Debt denomination in the model affects the extent to which the government uses inflation, default or bonds to finance its expenditures. With nominal debt, it uses inflation more readily since it can both raise seigniorage revenue and devalue the debt. Households holding bonds need to be compensated for expected inflation, which lowers the price of nominal bonds and curbs equilibrium borrowing by the government. Real debt provides less of an incentive for the government to inflate since it only generates seigniorage revenue. Real bond prices in turn only need to compensate households for default risk so the government can rely more heavily on bond finance than seigniorage. It will resort to inflationary finance only if the debt burden that needs to be financed is too high, bond revenues are too low because of default risk or it does not want to default instead.

In a simplified version of the model I characterize the trade off between inflation and default in partial equilibrium. I show that, for a given constant level of debt that is being rolled over each period, the government inflates and defaults relatively more when this debt is real than when it nominal to satisfy its budget constraint and maintain the optimal mix of inflation and default taxes. The intuition is that, when moving from a nominal debt to a real debt economy, higher default incentives hurt bond prices and thus revenue, while lower inflation incentives provide no offsetting boost to revenues via bond prices. On net, the government would be left with too little revenue and must raise additional revenue either via inflation, or borrowing which is associated with more inflation and default, in order to make up the shortfall. This simplified partial equilibrium version can feature large differences in inflation and default across the two economies even for the same level of debt.

I evaluate the quantitative effect of debt denomination on inflation and default in a stochastic general equilibrium version of the model that includes cash and credit consumption goods as well as labor income taxes. The nominal debt economy is calibrated to match the observed inflation and default rates in nominal debt issuing countries, and compared to an otherwise identical real debt economy. I find that in the real debt world the government optimally chooses higher inflation and default rates, as in the data. Quantitatively, the model captures the difference in inflation across the two debt regimes, and generates about two thirds of the difference in default rates.

The dynamics of the model shed light on the role of debt denomination in shaping equilibrium outcomes. For low levels of debt, a real debt regime is better able to contain inflation and default probabilities. This reflects the fact that incentives to inflate today are higher with nominal debt: it generates seigniorage and devalues the debt. For higher levels of debt, however, the picture switches. As the real debt economy enters the region of the state space where default risk is positive but finite, bond revenues fall since bond prices reflect the default risk, and the government begins to raise revenue through seigniorage. For the calibrated default cost that generates observed default frequencies in a nominal debt world, default risk is relatively sensitive to debt levels in the real debt economy beyond a certain level of debt. Inflation rates in the real debt economy match this rapid rise as the country substitutes bond finance for inflationary finance. The properties of the shock process together with the default cost ensure that the government in the real debt economy frequently visits this region of the state space where default risk and hence inflation are higher than in the nominal debt economy.

The model captures the comovement that is observed in the data between inflation and default - inflation tends to rise in the run up to default episodes. Simulated seigniorage and bond revenues as a percentage of GDP are realistic. In terms of debt levels, the real debt economy features higher debt to GDP ratios than the nominal debt economy, but the differences are modest compared to the differences in inflation and default rates. Default risk is important in generating the large differences in inflation rates that we see in the data. Absent default risk, the government in a real debt world accumulates substantially more debt than its nominal bond counterpart, and inflates at modestly higher rates, driven by the higher debt burden that it needs to finance. Issuing nominal debt is welfare improving, with small but positive lifetime consumption equivalent welfare gains of around 0.12%. These gains are the result of both lower average inflation and default costs, as well as lower volatility of allocations.

The paper highlights the importance of the connection between lack of commitment to monetary and fiscal policy. Real debt in the framework presented here does not remove the incentive to inflate. The paper emphasizes that real debt leads to worse outcomes in terms of countries' ability to manage debt crises. When the government cannot commit to either inflation or default, addressing the commitment problem on the fiscal front by issuing real debt can exacerbate the inflationary commitment problem.

Related Literature

The model is a monetary version of quantitative sovereign default models as in Arellano (2008). It shares with this literature that default is modeled as endogenous and dependent on fundamentals, and that governments lack commitment. I introduce costs of inflation and default in standard ways. Inflation is costly because of a cash in advance constraint on consumption as in Lucas and Stokey (1987) and Svensson (1985). Default is costly because it incurs a cost in terms of resources, akin to output costs used in many sovereign default studies.

The paper differs from existing papers in two key dimensions. First, it focuses on the difference between expropriation through inflation versus outright default as qualitatively different phenomena. Other studies restrict attention to inflation when analyzing the role of debt denomination (for example Diaz-Gimenez et al. (2008)), while the sovereign default literature predominantly assumes real, foreign currency external debt. Second, the paper distinguishes between the cost of inflation and debt denomination. In particular, even when bonds are real there is still an incentive to inflate in my framework. This corresponds to an economy where the government issues indexed debt but still has control over its own currency. Issuing real debt is not equated to dollarization or joining a monetary union.

Two papers that are closely related to mine are Martin (2009) and Diaz-Gimenez et al. (2008). The former studies the determination of nominal public debt levels in a setting without commitment. His application focuses on war finance in advanced economies. He does not consider real debt or default. The latter analyzes monetary policy under different debt denominations in an economy as in Nicolini (1998). The authors find that the welfare effect of nominal versus indexed debt are in general ambiguous and show how they depend on parameters, specifically the intertemporal elasticity of substitution. Both papers model money demand as arising from a cash in advance constraint on consumption, and both address lack of commitment on the part of the government, as does this paper. Neither considers the interaction of monetary policy with default which is a key focus here.

Domestic or nominal debt and self-fulfilling sovereign debt crises are the topic of a number of recent papers, including Aguiar et al. (2013), Lorenzoni and Werning (2013), Da-Rocha et al. (2013) and Araujo et al. (2013). They focus on self-fulfilling, expectations driven debt crises as in Calvo (1988) and Cole and Kehoe (2000) whereas I consider default driven by weak fundamentals. Another important difference is that these papers compare economies without any role for domestic monetary policy - a currency union of dollarization - with economies with nominal debt and monetary policy. I focus on an environment where money always plays a role and the country has control over its monetary policy; the issue of debt denomination is distinct from the choices of whether to relinquish control of the printing press. Other related papers that study sovereign default and foreign currency debt are Arellano and Heathcote (2010) in a model of dollarization and limited enforcement, and Gumus (2013) in a two-sector model and bonds that are either denominated in terms of tradables or nontradables. The latter paper finds in a result resembling the one in this paper that debt whose repayment value fluctuates with the state of the world (nontradable debt that is not subject to exchange rate fluctuations) yields better outcomes in terms of default rates and welfare.

Less closely related in terms of modeling approach, but related in terms of topic are numerous papers that address the benefits and costs of indexed versus nominal debt, including Missale (1997), Bohn (1990)who discusses the benefits of nominal debt in terms of making returns state contingent, Barro (1997) and Alfaro and Kanczuk (2010) who argue in favor of indexed debt (without considering explicit default as an option for the government. There is a large literature that explores optimal taxation, including through inflation and default, under full commitment, including in Chari and Kehoe (1999). The computation of Markov equilibria in macroeconomic dynamic models was first developed by Klein et al. (2008).

On the empirical side, there are a number of papers that study the currency composition of sovereign debt, as well as the connection between domestic default and inflation. Reinhart and Rogoff (2011) focus on domestic debt and default over a long period of time. They document that inflation and default episodes tend to occur together, that domestic default, even though less prevalent than external default, does occur with some frequency. Claessens et al. (2007) explore empirically the determinants of local currency debt as well as debt shares using non-publicly available data from the BIS. Cowan et al. (2006)construct a detailed debt database with a focus on Latin American countries, Guscina and Jeanne (2006) analyze debt composition for a subset of the countries I consider.

2 Nominal government debt in the data

In this section I will first document that the majority of emerging market government bond debt is denominated in nominal, local currencies, and second show that in these economies high nominal debt shares tend to be associated with low inflation and default rates.

2.1 Bond data

I construct local and foreign currency government bond debt estimates for a range of emerging market countries using Bloomberg bond-level data. The dataset contains all sovereign bond issues that were outstanding at some point between January 1, 1990 and December 31, 2012. For each issue, it includes information on the face value, the currency, the coupon structure, the maturity and issue date.

The countries I consider are a broad set of 27 emerging market countries, as classified

Table 1: Emerging bond market characteristics in 2012

	All	LatAm	Asia	Europe	Africa
Local currency	85.44	72.45	96.28	77.11	91.55
Domestic market	84.60	72.42	96.04	72.45	94.46
Zero coupon	18.05	28.05	11.83	12.38	35.63
Fixed rate	64.36	47.40	71.93	71.67	64.27
Pay at maturity	93.78	82.22	99.72	94.91	100.00
Inflation indexed	4.93	10.51	0.47	7.10	5.27
Defaulted/ Restructured	1.31	4.54	0.01	0.02	0.00

Percent. Bonds are valued using discounted US dollar face values. Percentages for regions are out of region totals.

by the IMF in 2012. They account for just over 80% of total emerging market GDP on average over the last two decades and and include the top 20 largest emerging market bond issuers.[2] The countries are not restricted to a particular continent and include Eastern European, Latin American, Asian and African economies. I include Korea which is classified as advanced according to the IMF. The set contains both defaulters and non-defaulters.[3]

To calculate debt stocks for each of these countries, I restrict attention to bonds with a simple payout structure. I only include bonds whose face value is paid back at maturity ("bullet" bonds rather than callable, sinkable or hybrid bonds with equity-like structure), and that have deterministic coupon payments (zero, fixed rate or step rate coupons). I also exclude defaulted bonds.

This is not restrictive as the vast majority of bonds do in fact have such a simple payout structure, as shown in Table (1). The table breaks the the dataset down by bond characteristics at the end of 2012.[4] It shows that across regions the majority of debt is in local currencies, issued on domestic markets, with either fixed or zero coupons and payable

[2]The top 5 as of 2012 are Brazil, China, India, Korea and Mexico.

[3]See a full list in the appendix. For some countries the time series starts later than 1990 because there are no bonds in the dataset early in the sample. See precise start dates in the appendix.

[4]These statistics are based on bonds valued as the discounted sum of their face value. I use this simple measure because it is easy to construct for all bonds, including the ones that I do not include in the calculation of the debt stock later on. It abstracts from coupon payments and assume that bonds that can be redeemed before maturity are never expected to be. Note that many measures of government debt stocks do in fact not include any coupon payments, including the external debt estimates by the World Bank WDI.

at maturity. Less than 5% of al debt is indexed, with the highest share in Brazil. Defaulted (and not restructured) bonds are a small fraction of the total.

2.2 Debt stocks

I construct debt stock estimates as the sum of discounted future payments over all outstanding bonds. I first calculate the value $b_{n,t}$ of each bond issue n at time t, as

$$b_{n,t} = \sum_{s=0}^{S} \frac{C_{n,s} + P_{n,s}}{(1+r)^s}$$

where $C_{n,s}$ and $P_{n,s}$ are coupon and principal payments, respectively, at time $t+s$, and S is the number of periods to maturity. I discount future payments at the constant annual interest rate $r = 4\%$.

If the bond is denominated in a currency other than US dollars, I convert the future payment stream using the exchange rate at time t. This means that the value of a bond can change over its lifetime not just because coupon payments are made, but also because of revaluation due to exchange rate movements. The principal and coupon values of indexed bonds are adjusted by multiplying by the so-called index factor - the change in the CPI between the issuance date and time t. The CPI is available monthly for most countries so I interpolate between dates.

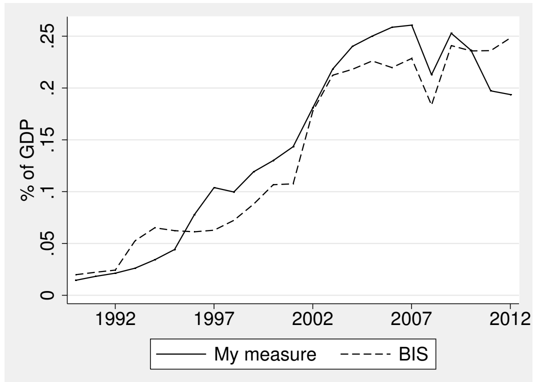
The stock of bond debt B_t is then calculated as the sum over all bonds outstanding at time t:

$$\begin{aligned} B_t &= \sum_{n=1}^{N} b_{n,t} \\ &= \sum_{n=1}^{N} \sum_{s=0}^{\bar{S}} \frac{C_{n,s} + P_{n,s}}{(1+r)^s} \end{aligned}$$

where N are the number of bond issues outstanding at time t, and $\bar{S}$ is the maximum number of periods left to maturity for all bonds. The frequency I choose is daily.

Figure (2.1) plots the aggregate debt stock for all countries in my sample over time as a fraction of aggregate GDP. This amounted to around 30% in 2012. In the Appendix, Figure (2.1) shows that total bond debt issued in my sample of emerging market countries was worth around 4 trillion current US$ in 2012 - much smaller than in advanced economies

Figure 2.1: Aggregate bond debt as a fraction of aggregate GDP



but growing over time.

In order to assess the accuracy of my estimates I compare them to aggregate data available from the BIS debt securities database. This database records quarterly general government debt securities outstanding for both domestic and international bonds from 1993Q3. The series for domestic securities is less complete - there are data for 16 of my countries and the series start late for some of them.

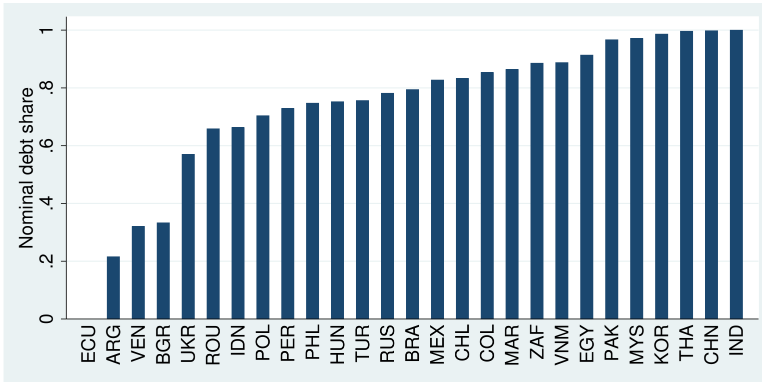
The sum of domestic and international debt from the BIS is plotted in the Figures alongside my estimates. The two series comove closely and the levels match up well. The uptick and subsequent drop around 1998/1999 in my series reflects the Russian default and increases in mainly local currency debt - one reason why this is not captured by the BIS is that the series for domestic debt does not start until 2005 for Russia.

The hump-shaped path of debt-to-GDP ratios in the early 2000s is driven by Latin American countries. All of them except Ecuador increased their debt stocks rapidly at the beginning of the millennium from very low levels, and experienced a sharp drop at the onset of the financial crisis, relative to GDP. Most countries saw their bond debt-to-GDP ratios fall around the time of the financial crisis in 2009.[5]

It is worth noting that the clear upward trend in bond debt levels is not exclusively driven by the move towards bond markets and away from bank debt finance. Gross general government debt according to the IMF WEO shows a similar pattern as the bond debt

[5]The gap towards the end of the sample between my and the BIS measure is driven by my estimates of Brazil's debt stock being too low especially for the last five years of the sample. I am working on improving this.

Figure 2.2: Nominal debt shares in 2012



measure shown here. On average over time, bond debt according to both my and the BIS measure constitutes around half of gross general government debt (which includes in addition liabilities such as social security and pensions), and this fraction actually falls slightly over time.

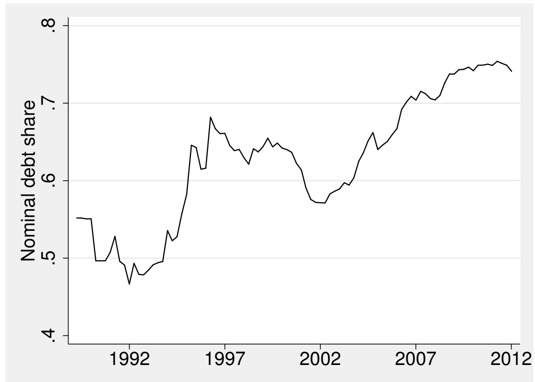
2.3 Nominal debt shares

I now turn to the question of how much emerging markets rely on foreign currency compared to local currency debt. I construct measures of the debt stock as above, separately for local currency and foreign currency denominated bonds. Figure (2.2) plots the resulting local currency debt share for each country at the end of 2012.

The Figure shows that the majority of emerging market debt is in local currencies. The unweighted average across countries in 2012 is 75%, a GDP-weighted average is slightly higher. All countries except four have local currency shares above 40%. Ecuador has a share of zero since it is dollarized, Argentina, Venezuela and Bulgaria are the other three countries that rely mostly on foreign currency denominated debt. At the other end of the spectrum, India has not issued a single foreign currency bond (in 2012 or the entire sample). Thailand, Korea, Malaysia, China and Pakistan all have local currency shares in excess of 95%.

Over time the share of debt issued in local currencies has increased substantially. This is shown in Figure (2.3) where we can see that the average local currency share stood at around 50% in 1990. The GDP-weighted average is lower at around 20% as the two largest

Figure 2.3: Nominal debt share over time, simple average across countries



countries in the sample in the early 1990s (in terms of constant US$ GDP), Mexico and Brazil, issued exclusively foreign currency debt.

Latin American countries generally have seen the largest shifts over time from foreign to local currency. In Asia local currency was more prevalent even early in the sample. Some countries have consistently issued mostly local currency debt: India, Korea, Malaysia, Pakistan, South Africa and Morocco.

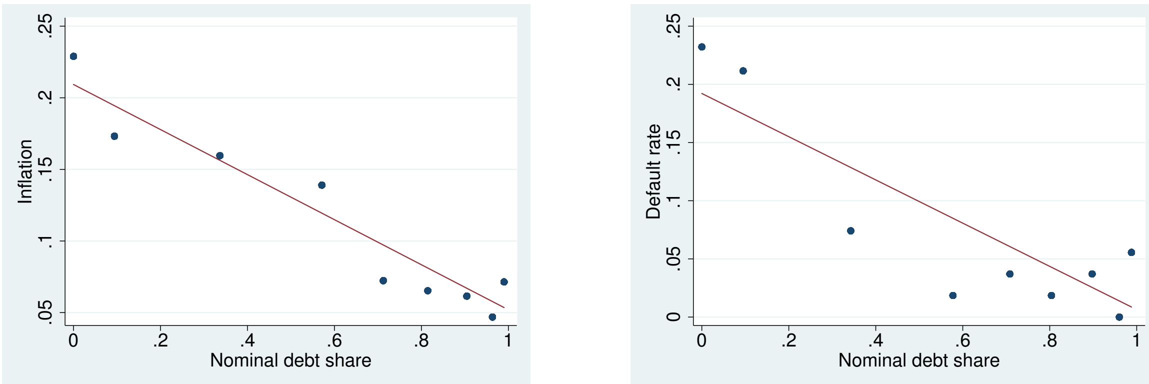
I cross-checked my results against data from the IMF's Public Sector Debt (PSD) database as far as possible. Data on the currency composition of public debt is available in the PSD for only 7 of my countries and a short sample. For Brazil, Indonesia, Peru, the Philippines and Poland the series start in 2010, for Mexico in 2005 and only for Hungary do they go back to 1999. Where available, the local currency shares from this data source match mine well.

2.4 Nominal debt, inflation and default rates

Next I document the correlation between high nominal debt shares and low inflation and default rates.

High inflation and default were widespread in the countries I am considering for at least part of the sample. Only 9 countries never defaulted between 1990 and 2012, with default defined by Standard & Poors sovereign ratings: China, Colombia, Egypt, Hungary, India, Korea, Malaysia and Thailand and Turkey. Turkey's sovereign crisis in 2001 crisis is not classified as a default by S&P.

Figure 2.4: Inflation rates and default against deciles of nominal debt shares



Pooled data. X-axis: Nominal debt share deciles, 642 obs per decile.

Y-axis left panel: Average inflation per decile. Y-axis right panel: #default obs / #obs per decile.

Inflation was very high in many of the countries in the sample: 11 of them, all in Latin America or Europe, recorded annual CPI inflation of at least 100% at some point.[6] Turkey, Venezuela, Ecuador and Romania had the highest median inflation over the whole period, all over 20%. Median inflation was the lowest in Morocco, Malaysia, Thailand and Korea with under 4%. Peru's median is actually also just below 4%, mainly because its hyperinflation ended just after the beginning of my sample.

I pool the data across countries and time, and split the observations into deciles by nominal debt share. I then compute average inflation and default rates for each decile. For inflation I exclude episodes of inflation that exceed 100% annually. Including these would make the results only stronger since the hyperinflation episodes are concentrated in the lower deciles of local currency shares.

The left panel of Figure (2.4) shows the resulting graph for inflation. There is a clear negative relationship between nominal debt shares and inflation. For the observations with the highest 10% of nominal debt shares, inflation was on average 8% annually. The lowest 10% in terms of nominal debt shares saw prices increase at a rate of 25%. The right panel of Figure (2.4) shows the analogous graph for default rates.[7] As in the case of inflation,

[6] Argentina, Brazil, Bulgaria, Ecuador, Peru, Poland, Romania, Russia, Turkey, Venezuela and the Ukraine.

[7] Default rates are defined as the total number of default states relative to the total number of states per decile. A default state is a country/year pair in which the country was in default - say Argentina in 2003. All states are all country/year pairs that fall in the given decile.

nominal debt debt shares and default rates are negatively correlated, with default rates for the highest nominal debt decile of around 7% compared with 27% for the set with the lowest local share of nominal debt.[8]

The correlations from both previous graphs are significant and robust to several modifications, including using median instead of mean inflation, period averages or period end inflation rates, and computing the deciles using local currency shares weighted by GDP. In addition, the Appendix contains the results from pooled and panel regressions of nominal debt shares on inflation and multinomial logistic regressions on default that control for a variety of factors. These controls include GDP, GDP per capita, debt levels, reserves, an index of democratic institutions, exchange rate regimes, independence of monetary authorities and inflation targeting dummies. Even after controlling for these factors, nominal debt shares are found to have a significant effect on inflation and the probability of default.

3 Model

In this section I will present a dynamic monetary model of sovereign borrowing without commitment, and use it to study the interaction between inflation, default and the denomination of debt. I will first discuss an economy where the government issues only nominal bonds, and then show what changes when instead it sells claims that are indexed to the price level.

Environment Time is discrete and infinite. The economy is populated by a representative agent with preferences over consumption and leisure, and a benevolent government that faces stochastic exogenous public consumption expenditures. Asset markets are incomplete with only money and one-period noncontingent bonds. The government has the monopoly over printing money and issuing bonds, and it lacks commitment to inflation, default and borrowing policies. When bonds are nominal it can devalue its debt through inflation, whereas real bonds are indexed to the price level. It can default on bonds of either denomination at any point. Inflation is costly because agents are subject to a cash-in-advance constraint on a subset of their consumption purchases, and default incurs a resource cost and temporary exclusion from credit markets.[9]

[8] See the Appendix for unpooled, raw scatter plots of inflation against nominal debt shares. Raw scatter plot of default are not informative obviously since it is a binary variable.

[9] I abstract from endogenous renegotiation and partial default in this paper.

Households The representative agent maximizes the expected discounted lifetime utility from consumption of cash and credit goods, and leisure. He enters every period with nominal money balances $\bar{m}_t$ and, if the government is not in default, sovereign bonds $\bar{B}_t$. Households split into a shopper who makes consumption purchases and a producer who transforms labor into output with a linear technology. He receives labor income net of taxes. I assume that the shopper must purchase the cash good c_{1t} using money balances that they hold at the start of the period

$$\bar{p}_t c_{1t} \leq \bar{m}_t \tag{3.1}$$

The can use bond holdings and labor income to finance credit good consumption. My timing assumption follows Svensson (1985) and Lucas and Stokey (1987) and implies that unexpected inflation is costly since agents are unable to adjust their balances after uncertainty is resolved. If the cash in advance constraint binds, agents spend all their money on cash consumption goods ((3.1) holds with equality). Higher than expected inflation then reduces the real purchasing power of the money that agents hold to buy c_{1t}.

Securities markets opens and households decide how to allocate receipts from cash god sales and invoices from credit good sales between money $\bar{m}_{t+1}$ and bonds $\bar{B}_{t+1}$ to carry into the next period. In equilibrium prices will adjust such that money and bond markets clear and households hold exactly as much money and bonds as the government issues.

The household therefore faces the budget constraint

$$\bar{p}_t c_{1t} + \bar{p}_t c_{2t} + \bar{m}_{t+1} + q_{nt}\bar{B}_{t+1} = (1-\tau)\bar{p}_t n_t + \bar{m}_t + \bar{B}_t$$

if the government is not in default and

$$\bar{p}_t c_{1t} + \bar{p}_t c_{2t} + \bar{m}_{t+1} = (1-\tau)\bar{p}_t n_t + \bar{m}_t$$

if it has defaulted. The labor tax rate τ is fixed exogenously. I introduce it since labor income taxes are an important source of government revenue empirically, but their determination is not the focus of this paper.

Following Cooley and Hansen (1991), in order to make the problem stationary I divide all nominal variables by the aggreagte money supply, that is $x \equiv \frac{\bar{x}}{\bar{M}}$, and define the money growth rate as $\mu_t \equiv \frac{\bar{M}_{t+1}}{\bar{M}_t} - 1$. Note that $\frac{\bar{x}_{t+1}}{\bar{M}_t} = \frac{\bar{x}_{t+1}}{\bar{M}_{t+1}}\frac{\bar{M}_{t+1}}{\bar{M}_t} = \frac{\bar{x}_{t+1}}{\bar{M}_{t+1}}(1+\mu_t)$. With this normalization, the households solves the problem

$$\max_{\{c_{1t},c_{2t},n_t,m_{t+1},B_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, 1-n_t)$$

subject to

$$p_t c_{1t} + p_t c_{2t} + (1+\mu_t)(m_{t+1} + q_{nt} B_{t+1}) = (1-\tau) p_t n_t + m_t + B_t$$

if the government is not in default,

$$p_t c_{1t} + p_t c_{2t} + (1+\mu_t) m_{t+1} = (1-\tau) p_t n_t + m_t$$

if it is in default, and the cash in advance constraint

$$p_t c_{1t} \leq m_t$$

as well as a nonnegativity constraint on money balances. I do not impose the constraint that the agent cannot be in debt ($\bar{B}_t < 0$) but in a distortionary equilibrium that features positive inflation and default rates they will lend to the government rather than the other way around. The utility function satisfies the standard properties.

Competitive Equilibrium and Asset Prices We can use the first order conditions of the household problem to characterize the competitive equilibrium in this economy. The cash-in-advance constraint implies

$$u_{1t} - u_{2t} \geq 0 \tag{3.2}$$

and as already mentioned in a competitive equilibrium where the cash in advance constraint binds such that $u_{1t} - u_{2t} > 0$, we have

$$c_{1t} = \frac{m_t}{p_t}$$

Labor is pinned down by

$$u_{lt} = (1-\tau) u_{2t}$$

We can derive expressions for asset prices from the household's problem. The first order conditions to his problem give rise to an equation for the money growth rate

$$\mu_t = \beta \mathbb{E}_t \left[\frac{u_{1,t+1}}{u_{2,t}} \frac{p_t}{p_{t+1}} \right] - 1 \tag{3.3}$$

Define consumer price inflation as $\pi_{t+1} \equiv \frac{p_{t+1}}{p_t} - 1$ and note that consumer price inflation and the money growth rate are related through

$$1 + \bar{\pi}_{t+1} = (1 + \pi_{t+1})(1 + \mu_t)$$

so we can alternatively express equation (3.3) as an equation for the price of money

$$1 = \beta \mathbb{E}_t \left[\frac{u_{1,t+1}}{u_{2,t}} \frac{1}{1 + \bar{\pi}_{t+1}} \right] \tag{3.4}$$

From equations (3.3), (3.4) and (3.2) we see that in a nonstochastic steady state monetary policy follows the Friedman rule - negative money growth and inflation at the rate of time preference - only if the cash in advance constraint does not bind. Outside the steady state, higher expected future inflation implies higher expected marginal utility of the cash good: If inflation is high, agents are cash-strapped and value cash good consumption relatively more.

Denote the states of the world in which the government chooses to default tomorrow by δ_{t+1}. The household problem implies the following epxression for the bond price:

$$q_{nt} = \beta \mathbb{E}_t \left[\frac{u_{2,t+1}}{u_{2,t}} \frac{1}{1 + \bar{\pi}_{t+1}} (1 - \delta_{t+1}) \right] \tag{3.5}$$

The price of nominal bonds reflects default, inflation and risk premia. The higher the risk of default δ_{t+1} and future inflation $\bar{\pi}_{t+1}$, the lower the price at which the bond sells today - agents demand a higher return to be compensated for the risk. The ratio of marginal utilities $\frac{u_{2,t+1}}{u_{2t}}$ reflects movements in the risk free rate. If this ratio is high, agents want to shift consumption from today to tomorrow by saving in bonds, driving up their price.

Government The government is benevolent and wants to finance exogenous public expenditures in the least distortionary way. It can print money, issue one-period noncontingent bonds, raise taxes, and default. It makes its decisions before the goods and securities markets for households open but after the shock has been realized. The government budget constraint if it repays is given by

$$\bar{M}_t + \bar{B}_t + \bar{p}_t g_t = \tau \bar{p}_t n_t + \bar{M}_{t+1} + q_{nt}\bar{B}_{t+1}$$

whereas in default they need to finance all of their expenditures with labor tax revenues and by printing money

$$\bar{M}_t + \bar{p}_t g_t^d = \tau \bar{p}_t n_t + \bar{M}_{t+1}$$

with $g^d \geq g$.[10] As in the household case, I normalize to get

$$1 + B_t + p_t g_t = \tau p_t n_t + (1 + \mu_t)(1 + q_{nt} B_{t+1})$$

and

$$1 + p_t g_t^d = \tau p_t n_t + (1 + \mu_t)$$

in repayment and default respectively.

I assume that the government is unable to commit to its policies and analyze Markov perfect equilibria of the economy throguhout the paper. Thus I assume that the government, when making its decisions, can only condition them on current fundamentals - debt B_t and the shock z_t - and take as given the policies implemented by future governments as well as the competitive equilibrium. In particular, it is unable to take into account that its policies today affected yesterday's outcomes. To make this concrete, the government would like to be able to promise to only inflate today and never again, but it cannot credibly commit to doing that. The government tomorrow only considers current and future tradeoffs and ignores promises made yesterday. The government todaty therefore knows and must take into account that inflating today will make it more expensive for the government tomorrow to borrow. Mechanically this can be seen from expressions (3.4) and (3.5), which enter the government budget constraint today but are functions of future inflation and default policies.

Real Debt In an economy where bonds are real instead of nominal, their value is fixed in units of the consumption good. Denote a claim to one unit of consumption by $b = \frac{\bar{B}}{\bar{p}}$ and its price by q_r. Then the household faces the following budget constraint if the government

[10] Whether it is households or the government paying the cost is not crucial.

does not default

$$\bar{p}_t c_{1t} + \bar{p}_t c_{2t} + \bar{m}_{t+1} + \bar{p}_t q_{rt} b_{t+1} = (1-\tau)\bar{p}_t n_t + \bar{m}_t + \bar{p}_t b_t$$

and the government budget constraint is

$$\bar{M}_t + \bar{p}_t \bar{b}_t + \bar{p}_t g_t = \tau \bar{p}_t n_t + \bar{M}_{t+1} + \bar{p} q_{rt} b_{t+1}$$

There are two main difference between issuing real compared to nominal debt. The first is that the value of oustanding liabilities is fixed in terms of consumption units for real debt. This can be seen by comparing the budget constraints of households and government in the two economies. An increase in the price level does not devalue the debt - the government cannot inflate it away.

The other difference arises from bond prices. We can derive bond prices in the same way as for the nominal debt economy to obtain

$$q_{rt} = \beta \mathbb{E}_t \left[\frac{u_{2,t+1}}{u_{2,t}} (1 - \delta_{t+1}) \right] \tag{3.6}$$

Compare this with (3.5): Because real debt cannot be inflated away, it does not carry an inflation premium and future inflation does not lower revenue from selling bonds today.

It is useful to note that bond prices can alternatively be written as follows, using the expression for the money growth rate (3.3) and inflation (3.4) which are the same in either economy:

$$q_{nt} = \mathbb{E}_t \left[\frac{u_{2,t+1}}{u_{1,t+1}} (1 - \delta_{t+1}) \right] \tag{3.7}$$

$$q_{rt} = \mathbb{E}_t \left[\frac{u_{2,t+1}}{u_{1,t+1}} (1 + \bar{\pi}_{t+1})(1 - \delta_{t+1}) \right] \tag{3.8}$$

This way of writing bond prices highlights the link between money and bonds as alternative assets in the model. For both real and nominal debt we see that the more the cash in advance constraint is expected to bind, that is the lower $\frac{u_{2,t+1}}{u_{1,t+1}}$, the lower bond prices are today. The reason is that money is in higher demand. Households would prefer to hold more money to avoid being short of cash in the next period.

In addition we can see that real bond prices are *higher* the higher inflation is tomorrow. This may seem counterintuitive, but is in fact also related to money and bonds being alternative assets. When inflation is expected to be high, real bonds are a more appealing investment than money since households know it cannot be devalued. This drives up bond prices relative to the price of money.

It is important to remember then that even if debt is real, the bond price is not independent of inflation because households are the holders of both money and bonds in the economy. The net effect of higher inflation on real bond prices is ambiguous: On the one hand, agents want to hold more money to avoid the cash in advance constraint binding which drives bond prices down, on the other, inflation makes real bonds the more attractive investment which drives their price up. In a nominal debt economy, bond prices unambiguously fall with higher inflation.

I will discuss the implications of this in the next sections after defining equilibria of both economies.

3.1 Recursive Equilibrium

I will state the problem recursively in order to define an equilibrium. The state of the nominal debt economy is B, the bond to money ratio, and the shock to governemt expenditure z. Assume $B \in \mathbb{B} \subset \mathbb{R}_+$ and $z \in \mathbb{Z} \subset \mathbb{R}$. In an equilibrium the government maximizes the representative households's utility subject to the government's budget constraint and the competitive equilibrium conditions. I am going to let B', p and d be the government's choice variables. The cash in advance constraint links prices to cash consumption, and the first order conditions pin down equilibrium money growth residually.[11] The commitment problem, as discussed above, means that borrowing today affects future price and default policies and that the government recognizes this, that is $p' = P(B', z')$ and $d' = D(B', z')$. Denote the exogenous probability of re-entering capital markets after a default by η.

Nominal Debt The government's option value of default is then given by

$$V(B, z) = \max\left\{V^r(B, z), dV^d(z)\right\} \tag{3.9}$$

[11] Alternatively and equivalently, think about the economy starting the period with a fixed money stock. Then the price level is pinned down by the cash in advance constraint during the goods market, and the difference between start and end of period money stocks pin down the money growth rate. The government decides on the money stock next period which shapes expectations for the price level tomorrow (see equation 3.4), and similarly for borrowing levels which affect default expectations.

where the value of repayment is

$$V^r(B,z) = \max_{p,B'} u(c_1,c_2,1-n) + \beta \int_{z'} V(B',z')dFz' \tag{3.10}$$

subject to

$$\begin{aligned}
1+B+pg(z) &= \tau pn + \left[1+\mu(p,B',z)\right]\left[1+q_n(p,B',z)B'\right] && (3.11)\\
c_1+c_2+g(z) &= n && (3.12)\\
c_1 &= \frac{1}{p} && (3.13)\\
u_c-u_l &\geq 0 && (3.14)\\
u_l &= (1-\tau)u_2 && (3.15)\\
\mu(p,B',z) &= \beta\int_{z'}\left[\frac{u_1'(P^r(B',z'),z')}{u_2(p,z)}\frac{p}{P^r(B',z')}\right]dF(z',z)-1 && (3.16)\\
q_n(p,B',z) &= \beta\int_{z'}\left[\frac{u_2'(P^r(B',z'),z')}{u_2(p,z)}\frac{p}{P^r(B',z')}(1-D(B',z'))\right]dF(z',z) && (3.17)
\end{aligned}$$

that is, the budget constraint, the resource constraint, the cash in advance constraint, the intratemporal competitive equilibrium condition pinning down labor, the cash in advance inequality condition from the household problem, and expressions for the money growth rate and bond prices.

In default the value is

$$V^d(z) = \max_p u(c_1,c_2,1-n) + \beta\int_{z'}\left[\eta V(0,z') + (1-\eta)V^d(z')\right]dFz' \tag{3.18}$$

subject to the analogous conditions

$$\begin{aligned}
1+pg^d(z) &= \tau pn + [1+\mu(p,0,z)] && (3.19)\\
c_1+c_2+g^d(z) &= n && (3.20)\\
c_1 &= \frac{1}{p} && (3.21)\\
u_l &= (1-\tau)u_2 && (3.22)\\
u_c-u_l &\geq 0 && (3.23)\\
\mu(p,B',z) &= \beta\int_{z'}\left[\frac{u_1'(P^d(z'),z')}{u_2(p,z)}\frac{p}{P^d(z')}\right]dF(z',z)-1 && (3.24)
\end{aligned}$$

with $g^d = g + h(g), h(g) \geq 0$. Note that both in repayment and default money growth rates depend on future prices, but the relevant price functions are different - in repayment, future price levels depend on the level of borrowing. In default - as I will show below - they reflect the probability of re-entering capital markets with zero debt. We can define an equilibrium price function $P(B, z)$ as

$$P(B,z) = \mathbb{1}\left(V^r(B,z) \geq V^d(z)\right) P^r(B,z) + \mathbb{1}\left(V^r(B,z) < V^d(z)\right) P^d(z) \tag{3.25}$$

and for default

$$D(B,z) = \mathbb{1}\left(V^r(B,z) < V^d(z)\right) \tag{3.26}$$

Definition 1. *Let $V : \mathbb{B} \times \mathbb{Z} \to \mathbb{R}, P : \mathbb{B} \times \mathbb{Z} \to \mathbb{R}_{++}, H : \mathbb{B} \times \mathbb{Z} \to \mathbb{B}$ and $D : \mathbb{B} \times \mathbb{Z} \to \{0, 1\}$. A Markov perfect equilibrium of the nominal debt economy are functions V, P, D, H as well as $c_1(B, z), c_2(B, z), n(B, z)$ and prices $\mu(P, H, z)$ and $q_n(P, H, z)$ that solve the government's problem (3.9) through (3.26) and where $B' = H(B)$.*

Real Debt Analogously to the nominal debt economy, the recursive problem of the government in a real debt economy is the following: Its option value to default is given by

$$V(b,z) = \max\left\{V^r(b,z), dV^d(z)\right\} \tag{3.27}$$

where the value of repayment is

$$V^r(b,z) = \max_{p,b'} u\left(c_1, c_2, 1-n\right) + \beta \int_{z'} V(b', z') dFz' \tag{3.28}$$

subject to

$$
\begin{aligned}
1 + pb + pg(z) &= \tau pn + \left[1 + \mu(p, b', z)\right] + q_r(p, B', z)pb' && (3.29)\\
c_1 + c_2 + g(z) &= n && (3.30)\\
c_1 &= \frac{1}{p} && (3.31)\\
u_l &= (1-\tau)u_2 && (3.32)\\
u_1 - u_2 &\geq 0 && (3.33)\\
\mu(p, b', z) &= \beta \int_{z'} \left[\frac{u_1'(P^r(b', z'), z')}{u_2(p, z)} \frac{p}{P^r(b', z')}\right] dF(z', z) - 1 && (3.34)\\
q_r(p, b', z) &= \beta \int_{z'} \left[\frac{u_2'(P^r(b', z'), z')}{u_2(p, z)} (1 - D(b', z'))\right] dF(z', z) && (3.35)
\end{aligned}
$$

and the value of default is

$$
V^d(z) = \max_p u\left(c_1, c_2, 1-n\right) + \beta \int_{z'} \left[\eta V(0, z') + (1-\eta)V^d(z')\right] dFz' \tag{3.36}
$$

subject to

$$
\begin{aligned}
1 + pg^d(z) &= \tau pn + \left[1 + \mu(p, B', z)\right] && (3.37)\\
c_1 + c_2 + g^d(z) &= n && (3.38)\\
c_1 &= \frac{1}{p} && (3.39)\\
u_l &= (1-\tau)u_2 && (3.40)\\
u_1 - u_2 &\geq 0 && (3.41)\\
\mu(p, B', z) &= \beta \int_{z'} \left[\frac{u_1'(P^d(z'), z')}{u_2(p, z)} \frac{p}{P^d(, z')}\right] dF(z', z) - 1 && (3.42)
\end{aligned}
$$

with $g^d = g + h(g), h(g) \geq 0$, and P and D satisfy

$$
P(B, z) = \mathbb{1}\left(V^r(B, z) \geq V^d(z)\right) P^r(B, z) + \mathbb{1}\left(V^r(B, z) < V^d(z)\right) P^d(z) \tag{3.43}
$$

$$
D(B, z) = \mathbb{1}\left(V^r(B, z) < V^d(z)\right) \tag{3.44}
$$

Definition 2. *Let* $V : \mathbb{B} \times \mathbb{Z} \to \mathbb{R}, P : \mathbb{B} \times \mathbb{Z} \to \mathbb{R}_{++}, H : \mathbb{B} \times \mathbb{Z} \to \mathbb{B}$ *and* $D : \mathbb{B} \times \mathbb{Z} \to \{0, 1\}$.

A Markov perfect equilibrium of the real debt economy are functions V, P, D, H as well as $c_1(B,z), c_2(B,z), n(B,z)$ and prices $\mu(P,H,z)$ and $q_n(P,H,z)$ that solve the government's problem (3.27) through (3.44) and where $b' = H(b)$.

Note that we can use the competitive equilibrium conditions for prices as well as the resource constraint to express the government's problem purely in terms of the state, choice for borrowing as well as current and future prices and default decisions (see the Appendix).

4 Inflation, Default and Borrowing in Equilibrium

In this section I analyze the channels through which the denomination of bonds affects the equilibrium of this model. We will see that the government in general prefers to spread the costs of inflation and default and thus uses both. Real debt on the one hand makes inflation less appealing because the debt burden is fixed in real terms and cannot be devalued. But on the other hand the government is less worried about causing inflation in the future because expected inflation does not depress bond prices when debt is real. In addition, the equilibrium level of borrowing affects the degree to which the government chooses to repudiate and monetize its liabilities. If the government borrows more then this adds to upward pressure on prices and default risk.

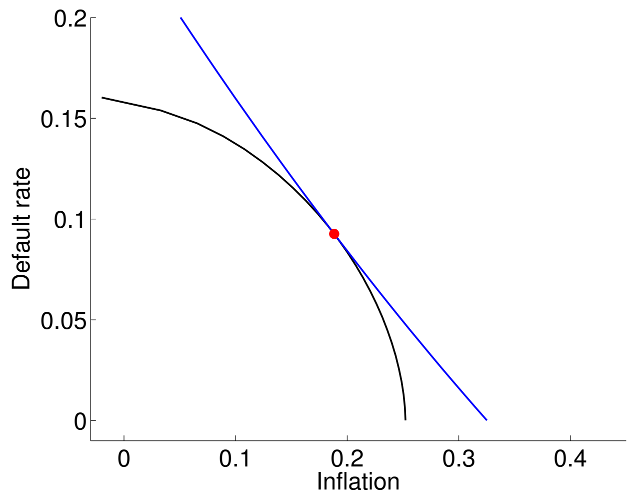
In order to draw out the main forces at work in the model I use a simplified version in this section. The key simplification is that I assume default is a continuous variable with the government choosing a default rate $d \in [0,1]$ each period. Defaulting incurs costs $t(d), t_d > 0, t_{dd} \geq 0$. There is no exclusion after default. With this assumption I can use first order conditions at an interior solution to the problem and characterize tradeoffs more precisely. I also abstract from uncertainty, labor income taxes and credit consumption goods. See the Appendix for a full definition of this simplified model.

Consider a nominal debt economy first. We can rewrite the government budget constraint purely in terms of allocation by using the competitive equilibrium conditions to substitute out prices. Then the constraints on the government's problem can be written as[12]

$$G^n(B,p,d,B',P(B'),D(B')) \equiv -u_2\left(\frac{1+(1-d)B}{p} + g + t(d)\right) + \beta\left(\frac{u_1' + u_2'(1-D(B'))B'}{P(B')}\right) = 0 \tag{4.1}$$

[12]See the Appendix for a full derivation

Figure 4.1: Static tradeoff between inflation and default



For a given B and income (borrowing B' with associated seigniorage and bond revenue)

and

$$F(p,d) \equiv u_1(p,d) - u_2(p,d) \geq 0$$

The first order conditions are

$$\begin{aligned} -\frac{u_c - u_l}{p^2} + \lambda G_p + \gamma F_p &= 0 \\ -u_l T_d + \lambda G_d + \gamma F_d &= 0 \\ \lambda \left(G_{b'} + G_P \frac{\partial P(B')}{\partial B'} + G_D \frac{\partial D(B')}{\partial B'} \right) + \beta \lambda' G_b' &= 0 \end{aligned} \tag{4.2}$$

At an interior solution where $F(p,d) > 0$ such that the cash in advance constraint is binding, the ratio of the first two conditions characterize the intratemporal optimal tradeoff between inflation and default

$$\frac{u_1 - u_2}{p^2(u_2 t_d)} = \frac{G_p^n}{G_d^n} \tag{4.3}$$

where

$$G_p^n = \frac{\partial G^n}{\partial p} = u_2 \frac{(1 + (1-d)B)}{p^2} + (u_{12} - u_{22}) \frac{1}{p^2} \left[\frac{(1 + (1-d)B)}{p} + g + t(d) \right] \tag{4.4}$$

$$G_d^n = \frac{\partial G^n}{\partial d} = u_2 \frac{B}{p} - u_2 t_d + u_{22} t_d \left[\frac{(1 + (1-d)B)}{p} + g + t(d) \right] \tag{4.5}$$

The left hand side of (4.3) is the relative cost of inflating compared to defaulting. The government is willing to trade off a fall in prices that reduces distortions by $\frac{u_1 - u_2}{p^2}$ for an increase in default that creates distortions $u_2 t_d$. Figure (4.1) illustrates this. It shows an "indifference curve" for default and inflation rates.[13] Along the curve, utility is held constant. The marginal rate of substitution between default and inflation, in other words the slope of this indifference curve, is given by the left hand side of equation (4.3).

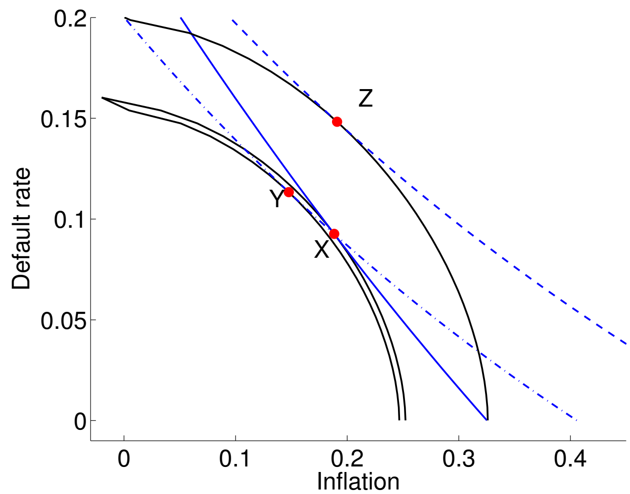
The right hand side of equation (4.3) represents the relative benefit to the government of devaluing through inflation compared to default in terms of relaxing its budget constraint. For a given level of income (the right hand side of (4.1)) and debt B, the picture plots the pairs of default and inflation rates that satisfy the government budget constraint. The slope of this line is equal to the negative of the right hand side of (4.3). At an optimum the government picks the lowest default and inflation pair that is feasible, at the point of tangency, the red dot in the picture.

The previous discussion focused purely on the intratemporal decision, holding borrowing and revenue fixed, but there are of course dynamic effects. It is not possible to solve analytically for the policy function for debt or the steady state, but we can use the intertemporal first order condition of the government's problem to gain insight into what drives equilibrium borrowing and how it affects the tradeoff between inflation and default. The intertemporal Euler condition for the government's problem, equation(4.2), describes the tradeoffs involved. It states that the government at an optimum equates the marginal benefit of borrowing today with the cost of repaying tomorrow. Borrowing more today lowers bond revenues but increases seigniorage revenue, which is reflected in the terms $G_P \frac{\partial P(B')}{\partial B'}$ and $G_D \frac{\partial D(B')}{\partial B'}$. The faster the sum of these terms falls with the level of debt, the less debt the government will accumulate in equilibrium. Note that we can derive analytical expressions for G_P and G_D but not for the partials with respect to the equilibrium policy functions, $\frac{\partial P(B')}{\partial B'}$ and $\frac{\partial D(B')}{\partial B'}$.[14] Numerically both prices and default rates are increasing

[13] The figure plots the tradeoffs in terms of inflation rather than price levels for expositional clarity. Analogous figures in terms of the price level look the same. See the Appendix for functional forms and parameters used in this example.

[14] An alternative way of expressing the Euler equation without substituting out prices is $\lambda\left(\frac{1+\mu}{p} q_n + \frac{\partial \mu}{\partial B'}\left(\frac{1}{p} + \frac{q_n B'}{p}\right) + \frac{\partial q_n}{\partial B'}\frac{(1+\mu)B'}{p}\right) = \beta\lambda' \frac{1-d'}{p'}$. The first term in brackets represents the direct additional revenue from selling a marginal unit of debt, q_n, in real terms. The second term captures additional seigniorage revenue. An increase in borrowing increases money growth and thus seigniorage revenue. The third term is the effect of increased borrowing on bond prices. Future expected inflation and default drive down bond prices and thus revenue today. The right hand side is the marginal cost of repaying the debt, in real terms.

Figure 4.2: Nominal and real debt: Substitution and income effects on inflation and default



Fixed $\frac{B}{p} = b$. X: Nominal bonds. Z: Real bonds. $X \rightarrow Y$: Substitution effect. $Y \rightarrow Z$: Income effect.

in debt levels, such that higher equilibrium debt levels translate to upward pressure on inflation and default.

In terms of Figure (4.1), higher borrowing and debt tend to shift the budget constraint out. The government partly finances a higher debt burden through inflation and default taxes since additional bond sales do not generate sufficient revenue.

4.1 Debt Denomination

When debt is real, incentives to inflate and default change. The relative cost of inflation compared to default is unaffected by the denomination of the debt since we are in a closed economy and debt does not affect utility directly via a resource constraint. What does change is the relative benefit from inflating, both today and tomorrow.

In terms of the *intra*temporal trade off that the government faces, inflation becomes less appealing when debt is real. We can see this by looking at the budget constraint for real debt

$$G(b, p, d, b', P(b'), D(b')) \equiv -u_2 \left(\frac{1}{p} + (1-d)b + g + t(d) \right) + \beta \left(\frac{u_1'}{P(b')} + u_2'(1 - D(b'))b' \right)$$

and comparing

$$\begin{aligned} G_p &= u_2\frac{1}{p^2} + (u_{12} - u_{22})\frac{1}{p^2}\left[\frac{1}{p} + (1-d)b + g + t(d)\right] \\ G_d &= u_2 b - u_2 t_d + u_{22} t_d\left[\frac{1}{p} + +(1-d)b + g + t(d)\right] \end{aligned}$$

with the corresponding expressions for nominal debt, equations (4.4) and (4.5). G_d is unchanged, but the second term in (4.4), $u_2\frac{(1-d)B}{p^2} > 0$ is missing in the analogous expression for real debt. Intuitively, the same drop in inflation now requires a smaller increase in default rates to satisfy the budget constraint, since the lower inflation did not increase the real value of outstanding bonds.

Figure (4.2) plots this along with the previous tradeoff from the nominal debt economy. It shows that the dashed budget constraint for the real debt economy has a flatter slope. Everything else equal, if the government in the nominal debt economy chooses a point like X, the government in the real debt economy with the same real debt burden and revenue would choose point Y - higher default and lower inflation.

How is revenue affected when debt is real instead of nominal? Recall that nominal bond prices in this model incorporate risk, default and inflation premia. In particular, the higher expected default and inflation tomorrow, the lower the bond price today since investors need to be compensated for the risk. This provides incentives for the government today to avoid inducing inflation tomorrow by borrowing more. The payout of real bonds on the other hand is not affected by inflation - there are no inflation premia in these bonds. As a result, the government today has less of a disincentive to cause inflation tomorrow by borrowing.

An alternative way of seeing this is to consider a steady state in which the real value of bond debt is the same across the two economies. Suppose the economy has been a nominal debt economy at a point like X in Figure (4.2), and suppose one morning all bond debt is suddenly indexed. As discssed above, holding revenue constant, the static effect is to increase default rates and lower inflation rates (moving to Y). We held revenue fixed, but the Markov government takes into account future policies which change with the denomination of the debt. In particular, with debt being real, a decrease in inflation rates will not be reflected in bond prices and thus give no boost to revenue, while an increase in default rates still hurts revenues. In terms of the budget constraint, as a result, without adjusting borrowing, revenues are too low. The government must either increase borrowing, or increase distortions today to make up the difference, and move to a point like Z in Figure (4.2). In the Appendix I prove under simplifying assumptions on utility and default costs, inflation and default are higher in a steady state where real debt levels are identical across a real and a nominal debt economy.

In this section we discussed how inflation, default rates and the level of borrowing are determined in the model. The government in general uses both default and inflation. When debt is real instead of nominal, there are two countervailing effects on equilibrium default and inflation: On the one hand, the government is less likely to use inflation because it is less effective at relaxing its budget constraint - real debt cannot be devalued through inflation. On the other hand, precisely because bond revenues are not hurt by future inflation, this encourages debt accumulation and for sufficiently high debt levels and default rates, the government may resort to higher inflation to generate seigniorage revenue after all. The next section explores which of these effects dominates quantitatively.

5 Quantitative Exercise

In this section I use the quantitative version of the model to evaluate which of the forces identified in the previous section dominate and to what extent the model can capture the data.

5.1 Parameters and Functional Forms

Utility is assumed to exhibit constant elasticity of substitution between cash and credit good consumption, and is separable in leisure:

$$u(c_1, c_2, 1-n) = \frac{\left((\alpha c_1^\rho + (1-\alpha)c_2^\rho)^{\frac{1}{\rho}}\right)^{1-\sigma}}{1-\sigma} + \nu\frac{(1-n)^{1-\theta}}{1-\theta}$$

Government spending in the model is given by $g = Ae^z$

$$z_{t+1} = \rho z_t + \epsilon_{t+1}, \epsilon \sim N(0, \sigma_\epsilon)$$

I calibrate the economy to match inflation and default rates in the average nominal debt issuing country in my sample. Average public consumption to GDP ratio in the data is 15%. Given a calibrate value for average hours worked, this pins down the value for A.

Instead of estimating the shock process based on the process for government spending for a particular country, I pick typical values for its persistence and volatility that are observed in a range of emerging market countries. I do not calibrate it to a particular time series since my question and calibration target is a cross-sectional observation. The average income tax rate is 15%. The long run growth rate of GDP is around 2% annually, so I choose $\beta = 0.98$ which implies an annual real risk free rate in the model economy of around 2%.

Government expenditures in default are parameterized as

$$\begin{aligned} g^d &= g + h(g) \\ &= g + \max\left\{0, \chi_1 \frac{\mathbb{E}[g]}{g} + \chi_2 \left(\frac{\mathbb{E}[g]}{g}\right)^2\right\} \end{aligned}$$

My specification implies that default is more expensive when government expenditures are low. This is analogous to papers in the sovereign default literature that assume that the cost of default is higher in good times. Without this assumption the value of default is too sensitive a function of the state and the model cannot generate large regions of the state space in which the country borrows with positive but finite default risk. I choose the probability of re-entry η such that the average default episode takes 4 years.

The remaining parameters are chosen to match moments from the data: Inflation, default rates, the debt-to-GDP ratio and ratio of cash to credit goods in the average nominal debt issuing country in my sample, as well as average hours worked of 0.3. Table (2)summarizes the chosen parameter values.

5.2 Simulation results

Table (3) presents statistics from the simulated model and compares them to the data. The targeted statistics for the nominal debt economy and the data counterparts are in the first two columns. The third and fourth column contain data and model statistics for real debt issuers.

As in the data, inflation and default rates are higher in the model when debt is real than when it is nominal. In terms of magnitudes the model slightly overpredicts the increase in inflation, and captures around two thirds of the change in default rates across debt regimes. Cash to credit goods ratios do not change much. The model predicts that debt levels in the real debt economy are higher than in the nominal debt economy but the difference is

Table 2: Parameters

Parameter	Value	Description/ Target
β	0.98	Discount factor
τ	0.10	Labor income tax rate
A	0.04	$\mathbb{E}[g]$
ρ	0.95	Persistence of AR(1) of g
σ_ϵ	0.0075	Volatility of AR(1) of of g
η	0.25	Probability of exclusion
ν, θ	1.93,8.51	Leisure level/ curvature
α	0.01	Cash good weight
ρ	-2.89	Cash - credit elasticity
χ_1,χ_2	-0.59,0.66	Default cost
σ	4.39	Risk aversion

Table 3: Data and simulated model statistics

	Nominal		Real	
	Data	Model	Data	Model
Inflation	5.5	6.7	21.1	23.0
Default probability	2.0	2.9	19.8	12.7
Cash-credit good ratio	30.4	30.4	21.9	29.8
Debt-to-GDP ratio	18.0	17.5	12.3	22.0

All statistics are in percent, model statistics are averages over periods of good credit standing, excluding the first 10 periods after the end of a default episode.

smaller across debt regimes than for inflation and default rates.

Figure 5.1 plots how the government accumulates debt over time in both economies. At a constant mean expenditure shock, the government begins to accumulate debt to finance expenditures. Inflation and default probabilities rise together with debt. In the real debt economy, the country begins to inflate later. At low debt levels expenditures plus debt service are low enough for the country to not resort to inflation. Only at sufficiently high debt levels inflation and default probabilities begin to rise.[15] An important feature of the grpah is that inflation and default probabilities rise rapidly in the real debt economy. Given the calibrated default process, default risk rises quickly beyond a given debt threshold and the country counteracts that with equally rapidly rising inflation. Debt accumulation levels off as default risk rises.

Figure 5.2 shows how seigniorage is used in both economies to offset the reduction in net bond revenues that occurs as bond prices fall. In the nominal debt economy this occurs sooner (at lower debt levels) and less suddenly. Even at low inflation rates bond prices fall with future expected inflation, lowering bond revenues and causing the country to switch to seigniorage. With real debt, the country uses bonds to generate revenue and only switches to seigniorage at higher debt levels.

To assess the welfare consequences of debt denomination policies I compute the consumption equivalent welfare gain of living in nominal debt economy. Specifically, I compute the fraction of aggregate lifetime consumption ω that agents born into the in real debt economy with no assets give up to live in nominal debt economy:

$$\mathbb{E}_0 \sum_t \beta^t u(\hat{c}_{n,t}, 1 - \hat{n}_{nt}) \quad = \quad \mathbb{E}_0 \sum_t \beta^t u(\hat{c}_{r,t}(1+\omega), 1 - \hat{n}_{rt})$$

which with power utility is the ω that solves

$$\omega \quad = \quad \left(\frac{\int V_r(0,z) dF(z)}{\int V_n(0,z) dF(z)} \right)^{\frac{1}{1-\sigma}} - 1$$

In my benchmark calibration the welfare gain from living in the nominal debt economy is 0.12% of lifetime consumption - small but positive.

In the real debt economy the rise in inflation is closely linked to default risk. In order to understand to what extent debt accumulation compared to default risk drive the result

[15] I plot realized inflation since that is my data counterpart, unexpected inflation looks qualitatively similar.

Figure 5.1: Debt, default and inflation dynamics in the model economies (percent)

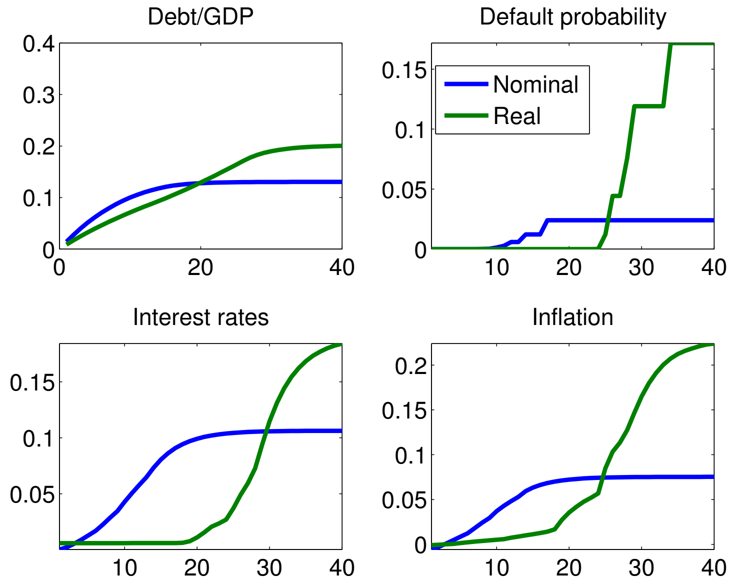


Figure 5.2: Sources of revenue in the model economies as a percent of GDP

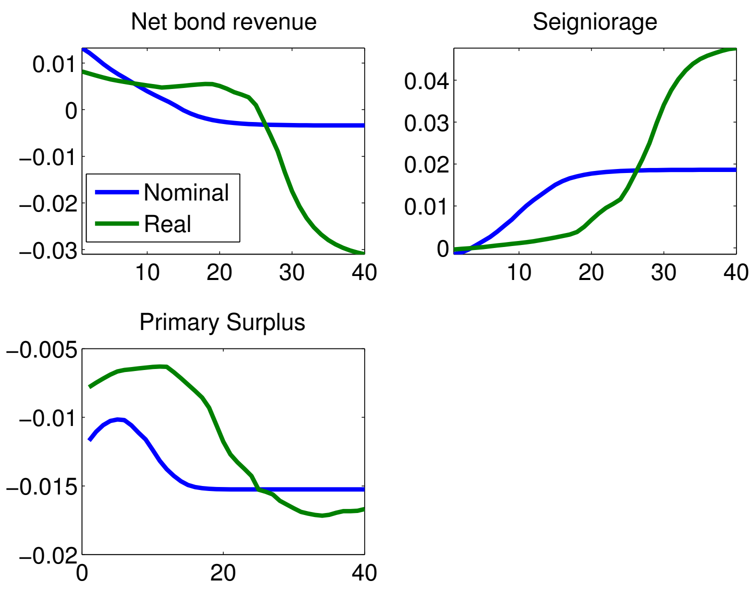


Table 4: Model simulations: Prohibitively costly default

	Baseline		No default	
	Nominal	Real	Nominal	Real
Inflation	6.7	23.0	15.2	28.7
Default probability	2.9	12.7	0.0	0.0
Debt-to-GDP ratio	17.5	22.0	26.0	58.1

All statistics are in percent, model statistics are averages over periods of good credit standing, excluding the first 10 periods after the end of a default episode.

that inflation is higher in a world with real debt I conduct a counterfactual experiment: What happens under both debt regimes when default is prohibitively costly such that it is not observed in equilibrium? Table 4 summarizes the main model statistics from this experiment along with the baseline results. It shows that without default risk, debt levels are substantially higher in the real debt economy while the difference across debt regimes in inflation is much smaller than in the baseline. This suggests that default risk is an important factor in generating the observed differences in inflation rates between nominal and real debt issuing countries. Without default risk differences in inflation rates are driven by debt accumulation with both the level and difference across debt regimes being counterfactually high. Default risk brings the model closer to the data.

6 Conclusion

In this paper I have studied sovereign inflation and default policies, and how these depend on debt denomination. In the data, countries that issue nominal debt tend to achieve lower inflation and default rates. A monetary model of sovereign borrowing and lack of commitment can account for this observation as a result of a change in debt denomination alone. Real debt lowers incentives to inflate today since it cannot be monetized, but it increases incentives to borrow and thus inflate tomorrow since real bond prices carry no inflation premia and inducing future inflation is less costly for the government. The second of these two forces dominates in a quantitative version of the model, leading to higher inflation and default rates, and lower welfare, in a real debt economy compared with an otherwise identical nominal debt economy.

The model highlights that even if bonds are real, money still plays a role and affects borrowing decisions. In particular, issuing real debt does not remove incentives to inflate. In terms of policy, the model suggests that issuing real debt in an attempt to address weak commitment by the policy maker may be short sighted and in fact exacerbate inflationary commitment problems. Considering lack of commitment as the only relevant factor, that is even absent institutional reforms, the model suggests that nominal debt issuance is a desirable policy.

In the paper I restrict attention to a setting where the debt denomination is a primitive and explore the consequences of debt regimes for government debt, default and inflation policies. It would be interesting to extend the paper in this dimension and explore whether and under what conditions the model can replicate the observed portfolios of nominal and real debt. Relatedly, studying the extent to which changes in institutional factors might affect these portfolios and changes in them over time is an interesting avenue of future research.

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Appendix A: Equilibrium - Simplified Model

Assume households have preferences over cash and credit good consumption and supply labor inelastically: $U = \sum_t \beta^t u(c_1, c_2)$. Assume the government can choose to default partially every period, where default incurs a resource cost $t(d)$ with $t(0) = 0$ and $t_d > 0$.

Nominal bonds

The household faces the following constraints

$$pc + (1+\mu)(m' + q_n B') = pn + m + (1-d)B$$

$$pc \leq m$$

The competitive equilibrium first order conditions imply

$$\begin{aligned} u_c - u_l &\geq 0 \\ \mu &= \beta \frac{u_1' p}{u_2 p'} - 1 \\ q_n &= \beta \frac{u_2'}{u_2} \frac{p}{p'(1+\mu)}(1-d') \\ &= \frac{u_2'}{u_1'}(1-d') \end{aligned}$$

Note that $c_1 + c_2 + g + t(d) = n = 1$.

The government faces the budget constraint

$$1 + (1-d)B + pg - pt(d) = (1+\mu)(1 + q_n B')$$

which can be rewritten in terms of allocations as

$$G(B, B', p, P(B'), d, D(B)) \equiv -u_2 \left[\frac{(1 + (1-d)B)}{p} + g + t(d) \right] + \beta \frac{(u_1' + u_2'(1 - D(B'))B')}{P(B')}$$

Definition 3. *Let $V : \mathbb{B} \to \mathbb{R}, P : \mathbb{B} \to \mathbb{R}_{++}, H : \mathbb{B} \to \mathbb{B}$ and $D : \mathbb{B} \to [0,1]$. A Markov perfect equilibrium of the simplified economy with nominal debt are functions V, P, D, H*

such that

$$V(B) = \max_{p,d,B'} u\left(\frac{1}{p}, 1 - \left(\frac{1}{p} + g + t(d)\right)\right) + \beta V(B')$$

subject to

$$\begin{aligned} G(B, B', p, P(B'), d, D(B)) &= 0 \\ F(p,d) \equiv u_1(p,d) - u_2(p,d) &\geq 0 \\ B' &= H(B) \end{aligned}$$

Real bonds

The household faces the following constraints

$$pc + (1+\mu)m' + q_r b' = pn + m + (1-d)b$$

$$pc \leq m$$

The competitive equilibrium first order conditions imply

$$\begin{aligned} u_c - u_l &\geq 0 \\ \mu &= \beta \frac{u_1' p}{u_2 p'} - 1 \\ q_r &= \beta \frac{u_2'}{u_2}(1-d') \\ &= \frac{u_2'}{u_1'}(1-d')\frac{p'(1+\mu)}{p} \end{aligned}$$

Note that $c_1 + c_2 + g + t(d) = n = 1$.

The government faces the budget constraint

$$1 + (1-d)b + pg - pt(d) = (1+\mu) + q_r b'$$

which can be rewritten in terms of allocations as

$$G(b, b', p, P(b'), d, D(b)) \equiv -u_2 \left[\frac{1}{p} + (1-d)b + g + t(d)\right] + \beta \left(\frac{u_1'}{P(b')} + u_2'(1 - D(b'))b'\right)$$

Definition 4. *Let $V : \mathbb{B} \to \mathbb{R}, P : \mathbb{B} \to \mathbb{R}_{++}, H : \mathbb{B} \to \mathbb{B}$ and $D : \mathbb{B} \to [0, 1]$. A Markov perfect equilibrium of the simplified economy with real debt are functions V, P, D, H such that*

$$V(b) \quad = \quad \max_{p,d,b'} u\left(\frac{1}{p}, 1 - \left(\frac{1}{p} + g + t(d)\right)\right) + \beta V(b')$$

subject to

$$\begin{aligned} G(b, b', p, P(b'), d, D(b)) &= 0 \\ F(p, d) \equiv u_1(p, d) - u_2(p, d) &\geq 0 \\ b' &= H(b) \end{aligned}$$

First Order Conditions

$$\begin{aligned} -\frac{u_c - u_l}{p^2} + \lambda G_p + \gamma F_p &= 0 \\ -u_l T_d + \lambda G_d + \gamma F_d &= 0 \\ \lambda\left(G_{B'} + G_P \frac{\partial P(B')}{\partial B'} + G_D \frac{\partial D(B')}{\partial B'}\right) + \beta\lambda' G_B' &= 0 \end{aligned}$$

where for nominal debt

$$\begin{aligned} G_p^N &= u_l \frac{(1 + (1-d)B)}{p^2} + (u_{lc} - u_{ll}) \frac{1}{p^2}\left[\frac{(1 + (1-d)B)}{p} + g + T(d)\right] \\ G_d^N &= u_l \frac{B}{p} - u_l T_d + u_{ll} T_d \left[\frac{(1 + (1-d)B)}{p} + g + T(d)\right] \end{aligned}$$

and

$$
\begin{aligned}
G_P &= -\frac{\beta}{P(B')}\left(\frac{u'_c}{P(B')}+\frac{u'_l(1-D(B'))B'}{P(B')}+\frac{u'_{cc}-u'_{cl}}{P(B')^2}+(1-D(B'))\frac{B'}{P(B')}\frac{(u'_{cl}-u'_{ll})}{P(B')}\right)\\
&= -\frac{\beta}{P(B')}\left(\frac{u'_c}{P(B')}+\frac{u'_l(1-D(B'))B'}{P(B')}\right.\\
&\quad +\frac{1}{P(B')}\left[\frac{u'_{cc}}{P(B')}-u'_{ll}(1-D(B'))\frac{B'}{P(B')}\right]\\
&\quad \left.+\frac{u'_{cl}}{P(B')}\left[-\frac{1}{P(B')}+(1-D(B'))\frac{B'}{P(B')}\right]\right)\\
G_D &= -\beta\frac{B'}{P(B')}(u'_l+u'_{ll}t_D(1-D(B')))
\end{aligned}
$$

while for real debt

$$
\begin{aligned}
G^R_p &= \frac{u_l}{p^2}+(u_{lc}-u_{ll})\frac{1}{p^2}\left[\frac{1}{p}+(1-d)b+g+T(d)\right]\\
G^R_d &= u_l b-u_l T_d+u_{ll}T_d\left[\frac{1}{p}+(1-d)b+g+T(d)\right]
\end{aligned}
$$

and

$$
\begin{aligned}
G_P &= -\frac{\beta}{P(b')}\left(\frac{u'_c}{P(b')}+\frac{u'_{cc}-u'_{cl}}{P(b')^2}+(1-D(b'))b'\frac{(u'_{cl}-u'_{ll})}{P(b')}\right)\\
&= -\frac{\beta}{P(b')}\left(\frac{u'_c}{P(b')}\right.\\
&\quad +\frac{1}{P(b')}\left[\frac{u'_{cc}}{P(b')}-u'_{ll}(1-D(b'))b'\right]\\
&\quad \left.+\frac{u'_{cl}}{P(b')}\left[-\frac{1}{P(b')}+(1-D(b'))b'\right]\right)\\
G_D &= -\beta b'(u'_l+u'_{ll}t_D(1-D(b')))
\end{aligned}
$$

Appendix B: Equilibrium - Full Model

The government's problem can be expressed purely in terms of allocations by substituting out competitive equilibrium conditions. An alternative equilibirum definition therefore is:

Definition 5. *Let $V:\mathbb{B}\times\mathbb{Z}\rightarrow\mathbb{R}$, $P:\mathbb{B}\times\mathbb{Z}\rightarrow\mathbb{R}_{++}$, $H:\mathbb{B}\times\mathbb{Z}\rightarrow\mathbb{B}$ and $D:\mathbb{B}\times\mathbb{Z}\rightarrow\{0,1\}$. Define also $\bar{z}(H(B))$ as the highest $z\in\mathbb{Z}$ for which the government chooses not to default.*

Then a Markov perfect equilibrium of the economy with nominal debt are funtions V, P, D, H *and a corresponding* $\bar{z}(H(B))$ *such that*

$$V(B,z) = \max\left\{V^r(B,z), V^d(z)\right\}$$

where the value of repayment is

$$V^r(B,z) = \max_{p,B'} u\left(\frac{1}{p}, n(p) - \frac{1}{p} - g, 1 - n(p)\right) + \beta \int_{z'} V(B',z')dF(z',z)$$

subject to

$$\begin{aligned}
u_2\left[\frac{(1+B)}{p} + g - n(p)\right] + u_l n(p) &= \beta\left(\mathbb{E}\left[\zeta_1(B',z')\right] + \mathbb{E}\left[\zeta_{2n}(B',z')\right]B'\right) \\
u_c - u_l &\geq 0 \\
u_l(n(p)) &= (1-\tau)u_2(p) \\
B' &= H(B)
\end{aligned}$$

where

$$\begin{aligned}
\mathbb{E}\left[\zeta_1(B',z')\right] &= \int_{z'\leq\bar{z}'} \frac{(u_1^r)'}{P^r(B',z')} f(z',z)dz' + \int_{z'>\bar{z}'} \frac{\left(u_1^d\right)'}{P^d(z')} f(z',z)dz' \qquad (6.1) \\
\mathbb{E}\left[\zeta_{2n}(B',z')\right] &= \int_{z'\leq\bar{z}'} \frac{(u_2^r)'}{P^r(B',z')} f(z',z)dz' + \int_{z'>\bar{z}'} \frac{\left(u_2^d\right)'}{P^d(z')} f(z',z)dz'
\end{aligned}$$

and the value of default is

$$V^d(z) = \max_{p} u\left(\frac{1}{p}, n(p) - \frac{1}{p} - g^d, 1 - n(p)\right) + \beta \int_{z'} \left[\eta V(0,z') + (1-\eta)V^d(z')\right] dF(z',z)$$

subject to

$$\begin{aligned}
u_2\left[\frac{1}{p} + g^d - n(p)\right] + u_l n(p) &= \beta\left(\eta\mathbb{E}\left[\zeta_1(0,z')\right] + (1-\eta)\mathbb{E}\left[\zeta_1^d(z')\right]\right) \\
u_c - u_l &\geq 0 \\
u_l(n(p)) &= (1-\tau)u_2(p) \\
B' &= H(B)
\end{aligned}$$

where ζ_1 is defined as in 6.1, and if the country stays in default

$$\mathbb{E}\left[\zeta_1^d(z')\right] = \int_{z'} \frac{(u_1^d)'}{P^d(z')} f(z',z)dz'$$

and P and D satisfy

$$P(B,z) = \mathbb{1}\left(V^r(B,z) \geq V^d(z)\right) P^r(B,z) + \mathbb{1}\left(V^r(B,z) < V^d(z)\right) P^d(z)$$

$$D(B,z) = \mathbb{1}\left(V^r(B,z) < V^d(z)\right)$$

Bond prices and money growth rates are given by, respectively

$$\begin{aligned} q_n(B',z) &= \frac{\mathbb{E}[\zeta_{2n}(B',z')]}{\mathbb{E}[\zeta_1(B',z')]} \\ q_r(b',z) &= \frac{\mathbb{E}[\zeta_{2r}(b',z')]}{\mathbb{E}[\zeta_1(b',z')]} \frac{(1+\mu(b',z))}{p} \\ &= \beta \frac{\mathbb{E}[\zeta_{2r}(b',z')]}{u_2} \\ \mu(B',z) &= \beta \frac{\mathbb{E}[\zeta_1(B',z')]}{\frac{u_2}{p}} - 1 \end{aligned}$$

Appendix D: Proofs

Claim. Consider the simple model (defined in Appendix A). Suppose utility is logarithmic in consumption and linear in leisure, and suppose default costs are linear $t(d) = \chi_1 d$. Consider a given steady state level of debt that is identical in the real and nominal debt economies: $b_r = b_n \equiv \frac{B}{p_n}$. Then inflation is higher in the nominal debt economy if and only if default rates are higher.

Proof. Suppose utility is log in consumption and linear in leisure: $u = \log c + \alpha l$ where $l = 1 - n$. Bond prices and money growth rates are determined in the competitive equilibrium and given by

$$\begin{aligned} \mu &= \beta \frac{u_1' p}{u_2 p'} - 1 \\ &= \beta \frac{p}{\alpha} \end{aligned}$$

and

$$
\begin{aligned}
q_n &= \beta \frac{u_2'(1-d')p}{u_2 p'(1+\mu)} \\
&= \frac{u_2'(1-d')}{u_1'} \\
&= \frac{\alpha(1-d')}{p'}
\end{aligned}
$$

The budget constraint for the government then is

$$
\frac{1}{p} + \frac{(1-d)B}{p} + g + t(d) = \frac{1+\mu}{p}\left(1 + q_n B'\right) \tag{6.2}
$$

$$
\frac{1}{p} + \frac{(1-d)B}{p} + g + t(d) = \frac{\beta}{\alpha}\left(1 + \frac{\alpha(1-d')B'}{p'}\right) \tag{6.3}
$$

The other constraint is the intratemporal condition relating default and inflation:

$$
\frac{u_1 - u_2}{p^2 u_2 t_d} = \frac{G_p}{G_d}
$$

which under our assumptions reduces to

$$
\frac{p-\alpha}{\alpha t_d} = \frac{1+(1-d)B}{-\frac{B}{p} + t_d} \tag{6.4}
$$

The analogous equations for a real debt economy are given by

$$
\begin{aligned}
q_r &= \beta \frac{u_2'(1-d')}{u_2} \\
&= \frac{u_2'(1-d')}{u_1'} \frac{(1+\mu)p'}{p} \\
&= \alpha(1-d')\frac{1+\mu}{p}
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{p} + (1-d)b + g + t(d) &= \frac{1+\mu}{p} + q_r b' \\
\frac{1}{p} + (1-d)b + g + t(d) &= \frac{\beta}{\alpha}\left(1 + \alpha(1-d')b'\right)
\end{aligned}
$$

and

$$\frac{p-\alpha}{\alpha t_d} = \frac{1}{-b+t_d}$$

Use subscripts to denote objects from the two different types of economy (n for nominal and r for real). Now consider the steady state of these economies with the same real value of debt taken as given, $b_r = b_n \equiv \frac{B}{p_n}$. Then we can solve two equations in two unknowns for equilibrium prices and default rates: For nominal debt

$$\begin{aligned}
\frac{1}{p_n} + (1-\beta)(1-d_n)b_n + t(d_n) &= \frac{\beta}{\alpha} - g \\
(p_n - \alpha)(t_{dn} - b_n) &= \alpha t_{dn}(1 + (1-d_n)p_n b_n)
\end{aligned}$$

and for real debt

$$\begin{aligned}
\frac{1}{p_r} + (1-\beta)(1-d_r)b_r + t(d_r) &= \frac{\beta}{\alpha} - g \\
(p_r - \alpha)(t_{dr} - b_r) &= \alpha t_{dr}
\end{aligned}$$

Suppose $p_r < p_n$. Then we must have

$$-d_n b + t(d_n) \quad < \quad -d_r b + t(d_r)$$

from the budget constraint. The net cost from inflating has to be lower in the nominal case. Under the assumption of linear default costs, this implies $d_r < d_n$ provided debt is not too low and the default cost is not too high, that is provided $b - \chi > 0$. This will hold in any equilibrium by the intratemporal FOC. Note that if $b - \chi < 0$, then the left hand side of the intratemporal condition is negative. But the right hand side is strictly positive for $d > 0$. $\square$

More generally, for different functional forms, provided the cost of defaulting is not disproportionately larger than the debt burden, we have that $d_r < d_n$. This is in general true the higher debt, and the lower the default cost.

Appendix E: Data

Sources and Coverage

All data run from 1990 through 2012 unless otherwise noted. Annual GDP data in current and constant US$ are from the WDI. The constant price series for Argentina ends in 2006. Monthly CPI inflation is from the IFS (except for the index for China which is from Global Financial Data). The series start later than January 1990 for Romania (Oct 1991), Ukraine (Jan 1993), Russia (Jan 1993) and Vietnam (Jan 96). Exchange rate data are from Global Financial Data. The series are daily except for Russia (weekly until Jan 1992) and Romania (monthly until Feb 1990). Bond data are from Bloomberg. The default data are based on Standard & Poors from 1990 to 2006, and extended thereafter through various other sources (news articles etc.). Quarterly data on aggregate government debt securities are from the BIS debt securities database. They are the sum of domestic and international debt securities; the international series is only available from 1993Q3. Annual general government debt is from the IMF World Economic Outlook.

Table 5: Countries, regions and first bond data observations in my sample

Latin America		Asia	
Argentina	15 Oct 1992	China	2 Jan 1990
Brazil	2 Jan 1990	Indonesia	30 Jul 1996
Chile	6 Dec 1994	India	2 Jan 1990
Colombia	8 May 1992	Korea	2 Jan 1990
Ecuador	8 May 1997	Malaysia	2 Jan 1990
Mexico	13 Mar 1991	Pakistan	6 Mar 1991
Peru	9 Mar 1998	Philippines	2 Jan 1990
Venezuela	15 Jan 1991	Thailand	2 Jan 1990
		Vietnam	28 Jul 2000

Europe		Africa	
Bulgaria	29 Jan 1998	Egypt	22 May 1997
Hungary	2 Jan 1990	Morocco	18 Mar 1993
Poland	12 Mar 1994	South Africa	2 Jan 1990
Romania	28 May 1996		
Russia	14 May 1993		
Turkey	2 Jan 1990		
Ukraine	21 Mar 1995		

Regression evidence

Regressions results documenting the negative relationship between nominal debt shares and inflation and default, respectively, are shown in Table (6). I estimate a linear model with country fixed effects for inflation rates

$$\log \text{inflation}_{it} \quad = \quad \alpha_i + \beta x_{it} + \epsilon_{it}$$

where x_{it} is the nominal debt share.

For default probabilities I use a simpe logit model

$$\text{logit} P(d_{it} = 1 | x_{it}, \zeta_i) = \alpha + \beta x_{it}$$

and also estimate two other versions where I allow for either random or fixed effects.

The tables show a significant negative relationship between nominal debt shares and both inflation and default. In terms of magnitude, the left panel shows that issuing nominal instead of real debt increases inflation by 1.5 log points. This corresponds roughly to a change from 5% annual inflation when all debt is nominal to around 20% when it is real. With year fixed effects the effect is somewhat smaller but still significant at 10%. The right panel of the Table shows that the probability of default is 90% lower for a country that issues all nominal instead of all real debt (I report the odds ratio of 0.118 in square brackets). Allowing for country random effects the magnitude of this effect shrinks to 50% but remains significant.

Table 6: Inflation and default probabilities

	Log inflation	
Nominal debt share	-1.435**	-0.896*
	(0.580)	(0.473)
N	6220	6220
adj. R^2	0.434	0.570
Time FE	No	Yes
Country	FE	FE

	Default		
Nominal debt share	-2.138***	-0.675***	-0.649***
	(0.621)	(0.205)	(0.205)
	[0.118]	[0.509]	[0.523]
N	6418	6418	2663
Country	-	RE	FE

Standard errors in parentheses * $p<0.10$ ** $p<0.05$ *** $p<0.01$ Odds ratios in square brackets

Additional Figures

Figure A.1: Aggregate bond debt in my sample of countries

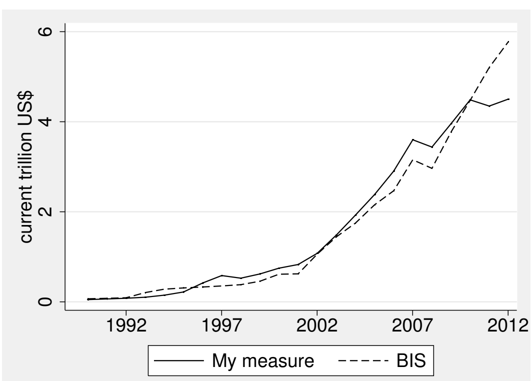

Figure A.2: Raw scatter plot of inflation against nominal debt share