Wage Distribution with a Two-sided Job Auction (preliminary and incomplete)

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Abstract

We derive a wage distribution in a model where homogenous workers and homogenous firms can send wage offers to each other. A static model and two dynamic models are considered. We solve the mixed strategies for wage offers and demands and for the fractions of firms and workers engaged in sending or receiving offers. The density function for realised wages is low and increasing at small wages, low and decreasing at high wages, and high and u-shaped at middle-range wages. The mixed strategy equilibrium is evolutionarily stable and utilitywise equivalent to auction where the number of competitors is known.

Theme: Microeconomics of unemployment

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1 Introduction

Labour markets constitute such a large part of the economy, and for many people leisure is the only significant endowment, that issues of labour economics can rarely be meaningfully addressed in partial equilibrium models. There are roughly two types of general equilibrium models used in modelling labour markets. One is a search model pioneered by Mortensen and Pissarides (e.g. Pissarides, 2000) where unemployed and firms are involved in a time consuming activity of looking for each other. In these models the meetings are pairwise, and wages are determined using the Nash bargaining solution or some analogous procedure. The central concept is so called matching function which tells how fast the parties find each other.

In the other branch of models the meetings of the unemployed and vacancies are governed by an urn-ball matching where the urns are contacted by the balls (see for example Montgomery, 1991). The advantage of this approach is that the matching function can be determined endogenously, and that it makes multiple meetings possible. In the end, it is clear that whatever one can do using the search models, one can also do using the urn-ball models, and with the latter ones one can do much more. To give a concrete example, it is relatively straightforward to consider a situation where vacancies post wages that are observed by the unemployed who strategically decide which vacancy to contact based on the observed wage offers. This is practically impossible in the search models. Against this background it is somewhat a mystery to us why search models are still used.

There are several empirical findings that are hard to come by in theoretical models. One of these is the empirical wage distribution. A typical wage distribution is hump-shaped and right skewed (DiNardo, Fortin and Lemieux, 1996). The non-degenerate wage distribution holds for observationally identical workers, too. Wage distributions have been generated by search models with varying results. In Burdett and Mortensen (1998), workers receive wage offers from employers at an exogenous rate, and workers search also on the job. With identical workers and identical firms, the wage offer density function is increasing. The wage offer density can be declining only if offers are made by firms that differ in productivity, or if workers are heterogenous in productivity There are also some articles that generate distributions of wages that more closely resemble the observed ones. To get this one must assume heterogeneity of workers or firms and some special features of the matching function. These features are, of course, not derivable from the basics of the model but are just assumed. Mortensen (2000) considers an on-the-job search model where workers receive wage offers from firms that can make match-specific investments after the firm and worker have met. The firms are heterogenous ex post with respect to the amount of capital. Firms who offer higher wages invest more in match-specific capital, because workers with higher wages have a lower probability to quit. The resulting wage offer density can be increasing, decreasing, or hump-shaped. However, for the density to be hump-shaped, it is required that the production function is of Cobb-Douglas type, and that the parameter of the production function and the exogenous reservation wage fall in certain limits. Bontemps, Robin and van den Berg (2000) allow for firms to have different technologies, and they show that a suitable distribution of employer productivity can lead to a hump-shaped distribution for wages.

In this article we demonstrate that one can, with very simple economic reasoning, generate a significant improvement to the wage distribution when one uses an urn-ball model. It also becomes clear that one cannot use the ideas of this article if one sticks to the search models. Another important assumption is that all firms as well as all workers are homogenous. We get three kind of results in this article. First, we generate a wage distribution that is first increasing, in the end decreasing, and u-shaped in the middle. Thus, we do not get exactly the wage distribution observed empirically but one that still has several desirable features. Second, we show that there are three possible equilibria in our model, and of these three only one features an interesting wage distribution. But this is the unique evolutionarily stable equilibrium. Most of the literature has focused on one of the non-stable equilibria. Third, in our model we assume that when several workers contact an employer, or the other way round, they do not know how many workers happen to contact the particular employer. This means that when workers make their wage demands, they must use a mixed strategy in equilibrium. We derive the mixed strategy explicitly and show that in utility terms it is equivalent to a mechanism where the workers know the number of their competitors, and wages are determined in an auction. Kultti (1999) shows the equivalence between such auctions and posted prices; we thus know that all the three mechanisms are equivalent in utility terms.

We describe the general idea of the model in Section 2. We solve the model for a static case in Section 3. Section 4 concludes. In Appendix 1 we present a dynamic steady state case which yields almost identical results to the static model when we assume that vacancies and unemployed that are matched disappear from the economy and are replaced by identical agents. In Appendix 2 we solve a full general equilibrium model where the number of workers and firms is given; some of them are matched while the others are either unemployed or vacancies. The most important figures are presented after the references.

2 The Model

In the static model, the measure of identical job seekers is u and the measure of identical vacancies is v. There are two submarkets. Fraction $x \in [0,1]$ of workers and fraction $y \in [0,1]$ of vacancies are in the 'vacancy market'. Each job seeker chooses randomly one of the yv firms and sends an application accompanied with a wage ask w. The workers do not know which firms the other workers apply to, neither do they know their wage demands. Each firm that has received at least one application hires the worker who has asked the lowest wage. We could say that in the vacancy maket, firms stay (or wait), and workers move. In the 'worker market', each of the $(1-y)\,v$ vacancies sends an offer w (moves) to one of the $(1-x)\,u$ workers. The vacancies do not know how much the other vacancies offer and to whom. A worker then chooses to work in the firm that has offered the highest wage. A matched firm-worker pair produces output of size unity. Utilities are linear such that a worker's utility is w, and firm's utility is $1-w$. Unmatched agents get zero utility. We solve the equilibrium fractions x and y as functions of u/v, and the distributions of wage offers and realised wages.

The first dynamic model is otherwise the same as the static model, except that the vacancies and unemployed that are matched exit from the economy and produce together an output of value of one. These agents are replaced by identical yet unmatched agents. In the second dynamic model, the total numbers of firms and workers are fixed. Some firms and workers are unmatched and they are either sending applications or offers, or they are waiting for them as in the two models described above. The rest of the agents are in bilateral matches producing an output of unity each period until the relationship breaks down which happens with probablity b each period. After a breakdown the agents start searching for partners again.

3 Distributions of Wage Demands and Offers in a Static Model

Case 1: Firms send offers to workers

In this market each firm sends a wage offer to one worker. The Poisson parameter in this market is ϕ (If all workers and firms are in this market, then $\phi = v/u$.) Let E_s be the utility of a firm that offers wage w. Firm use a mixed strategy with cumulative distribution function $H(w)$ with support $[b, B]$. The utility of a firm is

$$\begin{aligned} E_s &= e^{-\phi}(1-w) + \phi e^{-\phi}\left(1-(w)\right)H(w) + ... + \frac{\phi^k e^{-\phi}}{k!}(1-w)\left(H(w)\right)^k + ... \quad (1) \\ &= (1-w)e^{-\phi(1-H(w))}. \end{aligned}$$

The mixed strategy gives $(1-b)e^{-\phi(1-H(b))} = (1-B)e^{-\phi(1-H(B))}$. The lowest offer b equals zero, and using $b = 0$, $H(b) = 0$ and $H(B) = 1$, the upper limit of the wage offers is $B = 1 - e^{-\phi}$. A firm's utility is therefore

$$E_s = e^{-\phi}. \quad (2)$$

Using $E_s = (1-w)e^{-\phi(1-H(w))} = e^{-\phi}$ we get

$$H(w) = -\frac{1}{\phi}\ln(1-w) \quad (3)$$

with support $w \in \left[0, 1 - e^{-\phi}\right]$. The density function is

$$h\left(w\right) \equiv H'(w) = \frac{1}{\phi(1-w)}, \tag{4}$$

which is increasing in w.

Case 2: Workers send applications to firms

Let U_s be the utility of a job seeker who sends an application with wage ask w. Workers use a mixed strategy with cumulative distribution function $F(w)$ with support $[a, A]$. Let θ the appropriate Poisson parameter that governs the meeting probabilities (If all workers and firms are in this market, then $\theta = u/v$.) We have

$$\begin{aligned} U_s &= e^{-\theta} w + \theta e^{-\theta}\left(1 - F(w)\right) w + ... + \frac{\theta^k e^{-\theta}}{k!}\left(1 - F(w)\right)^k w + ... \\ &= w e^{-\theta}\left[1 + \sum_{k=1}^{\infty} \frac{\theta^k}{k!}\left(1 - F(w)\right)^k\right] \\ &= w e^{-\theta} \sum_{k=0}^{\infty} \frac{\theta^k \left(1 - F(w)\right)^k}{k!} \\ &= w e^{-\theta F(w)}. \end{aligned} \tag{5}$$

Worker's utility is the same for all $w \in [a, A]$, especially $ae^{-\theta F(a)} = Ae^{-\theta F(A)}$. Clearly, $A = 1$ because the probability that the worker in question is the only applicant is positive. Then $ae^{-\theta F(a)} = e^{-\theta} \Rightarrow a = e^{-\theta}$, and

$$U_s = e^{-\theta}. \tag{6}$$

Next we solve $F(w)$. We have $U_s = e^{-\theta} = we^{-\theta F(w)}$. Taking logarithms results in $\ln e^{-\theta F(w)} = \ln e^{-\theta} - \ln w \Leftrightarrow \theta F(w) = \theta + \ln w$, and the resulting distribution function is

$$F(w) = 1 + \frac{\ln w}{\theta}, \tag{7}$$

with support $w \in \left[e^{-\theta}, 1\right]$. The density function is

$$f\left(w\right) \equiv F'(w) = \frac{1}{\theta w}, \tag{8}$$

which is decreasing in w.

4 Distributions of Realised Wages in Static Model

Firms and workers draw their wage offers and asks from distributions $H(w)$ and $F(w)$ which are unobserved. The realised distributions are $G(w)$ in a market where workers send applications and $M(w)$ in the other market. In the market where firms send offers the realized wage distribution is

$$M(w) = \frac{\sum_{k=1}^{\infty} \frac{\phi^k e^{-\phi}}{k!} H^k(w)}{1-e^{-\phi}} \tag{9}$$

$$= \frac{e^{-\phi(1-H(w))} - e^{-\phi}}{1-e^{-\phi}}$$

$$= \frac{e^{-\phi}\left(\frac{1}{1-w} - 1\right)}{1-e^{-\phi}}.$$

The density function is

$$m(w) = M'(w) = \frac{e^{-\phi}}{\left(1-e^{-\phi}\right)(1-w)^2}. \tag{10}$$

Let us next consider the market where workers send applications. The probability that the lowest wage ask received by a firm is at most w equals

$$G(w) = \frac{\sum_{k=1}^{\infty} \frac{\theta^k e^{-\theta}}{k!}\left[1-(1-F(w))^k\right]}{1-e^{-\theta}} \tag{11}$$

$$= \frac{1-e^{-\theta} - e^{-\theta}\sum_{k=1}^{\infty} \frac{\theta^k (1-F(w))^k}{k!}}{1-e^{-\theta}}$$

$$= \frac{1-e^{-\theta} - e^{-\theta}\left(e^{\theta(1-F(w))} - 1\right)}{1-e^{-\theta}}$$

$$= \frac{1-e^{-\theta F(w)}}{1-e^{-\theta}},$$

where the denominator $1-e^{-\theta}$ conditions for receiving at least one application. Using $F(w) = 1 + \frac{\ln w}{\theta}$ we end up with

$$G(w) = \frac{1-\frac{e^{-\theta}}{w}}{1-e^{-\theta}}. \tag{12}$$

The density function is

$$g(w) \equiv G'(w) = \frac{e^{-\theta}}{\left(1 - e^{-\theta}\right) w^2}. \tag{13}$$

Realised aggregate wage distribution

The realised aggregate density function of wages, $r(w)$ is a weighted combination of densities $g(w)$ and $m(w)$ as follows:

1) $r(w) = \dfrac{m(w)(1-x)u}{(1-x)u+yv}$ if $w \in \left[0, e^{-\theta}\right]$,

2) $r(w) = \dfrac{m(w)(1-x)u + g(w)yv}{(1-x)u+yv}$ if $w \in \left[e^{-\theta}, 1-e^{-\phi}\right]$,

3) $r(w) = \dfrac{g(w)yv}{(1-x)u+yv}$ if $w \in \left[1-e^{-\phi}, 1\right]$.

For $w < e^{-\theta}$, only firms send offers, and there are $(1-x)u$ workers who receive them. For $w > 1-e^{-\phi}$, only workers send offers to yv firms. For middle-range wages, $(1-y)v$ firms send offers to $(1-x)u$ workers and xu workers send wage demands to yv firms. We have yet to determine the equilibrium values of x and y, which are determined by indifference conditions for firms and workers.

5 Utilities

In the market where workers send applications, a worker's utility is $U_s = e^{-\theta}$. A firm receives k applications, and the probability that the wage asked by worker i is the lowest is $(1-F(w))^{k-1}$. A firm's expected utility in this market is equal to E_r:

$$E_r = \int_a^A \sum_{k=0}^{\infty} \frac{\theta^k e^{-\theta}}{k!} f(w) k \left(1-F(w)\right)^{k-1} (1-w) dw \tag{14}$$

$$= \sum_{k=1}^{\infty} \frac{\theta^k e^{-\theta}}{k!} \left[-\left(1-F(A)\right)^k + \left(1-F(a)\right)^k \right]$$

$$- \int_a^A \sum_{k=1}^{\infty} \frac{\theta^k e^{-\theta}}{k!} f(w) k \left(1-F(w)\right)^{k-1} w dw$$

$$= 1 - e^{-\theta} - \int_a^A \sum_{k=1}^{\infty} \frac{\theta^k e^{-\theta}}{k!} \frac{1}{\theta w} k \left(\frac{\ln w^{-1}}{\theta} \right)^{k-1} w dw$$

$$= 1 - e^{-\theta} - \int_a^A \sum_{k=0}^{\infty} \frac{\left(\ln w^{-1}\right)^k}{k!} e^{-\theta} dw$$

$$= 1 - e^{-\theta} - \int_a^A \frac{e^{-\theta}}{w} dw$$

$$= 1 - e^{-\theta} - e^{-\theta}\left(\ln 1 - \ln e^{-\theta}\right)$$

$$= 1 - e^{-\theta} - \theta e^{-\theta}.$$

In a market where firms send offers, a firm's utility is $E_s = e^{-\phi}$. A worker receives k offers, and the probability that the wage offered by firm i is the highest is $(H(w))^{k-1}$. A worker's expected utility in this market is equal to U_r:

$$U_r = \int_b^B \sum_{k=1}^{\infty} \frac{\phi^k e^{-\phi}}{k!} h(w) k \left(H(w)\right)^{k-1} w dw \tag{15}$$

$$= \int_b^B \sum_{k=1}^{\infty} \frac{\phi^k e^{-\phi}}{k!} \frac{1}{\phi(1-w)} k \left(\frac{\ln(1-w)^{-1}}{\phi}\right)^{k-1} w dw$$

$$= \int_b^B \sum_{k=1}^{\infty} \frac{\left(\ln(1-w)^{-1}\right)^{k-1}}{(k-1)!} e^{-\phi} \frac{w}{1-w} dw$$

$$= \int_b^B \sum_{k=0}^{\infty} \frac{\left(\ln(1-w)^{-1}\right)^{k}}{k!} e^{-\phi} \frac{w}{1-w} dw$$

$$= \int_b^B e^{-\phi} e^{\ln(1-w)^{-1}} \frac{w}{1-w} dw$$

$$= e^{-\phi} \int_b^B \frac{w}{(1-w)^2} dw$$

$$= e^{-\phi} \int_b^B \left(\frac{1}{(1-w)^2} - \frac{1}{1-w}\right) dw$$

$$= e^{-\phi}\left[\ln(1-B) + \frac{1}{1-B} - \ln(1-b) - \frac{1}{1-b}\right]$$

$$= 1 - e^{-\phi} - \phi e^{-\phi}.$$

6 Equilibrium

In an equilibrium where two markets co-exist, the utility of a firm that sends offers is the same as the utility of a firm who receives wage demands from workers. The same equivalence condition holds for workers, too. That is, we have $E_s = E_r$ and $U_s = U_r$, and inserting the utilities derived above yields

$$\begin{aligned} e^{-\phi} &= 1 - e^{-\theta} - \theta e^{-\theta}, & (16) \\ e^{-\theta} &= 1 - e^{-\phi} - \phi e^{-\phi}. & (17) \end{aligned}$$

Firms's equilibrium condition (16) is named EE, and worker's equilibrium condition (17) is named WE. When both conditions hold we have $\theta e^{-\theta} = \phi e^{-\phi}$, and after substitution we get

$$\begin{aligned} \phi &= \frac{\theta e^{-\theta}}{1 - e^{-\theta} - \theta e^{-\theta}}, & (18) \\ \theta &= \frac{\phi e^{-\phi}}{1 - e^{-\phi} - \phi e^{-\phi}}. & (19) \end{aligned}$$

Using (18) and (19) in $\theta e^{-\theta} = \phi e^{-\phi}$ results in

$$\begin{aligned} 1 - e^{-\theta} - \theta e^{-\theta} - e^{\frac{-\theta e^{-\theta}}{1 - e^{-\theta} - \theta e^{-\theta}}} &= 0, & (20) \\ 1 - e^{-\phi} - \phi e^{-\phi} - e^{\frac{-\phi e^{-\phi}}{1 - e^{-\phi} - \phi e^{-\phi}}} &= 0. & (21) \end{aligned}$$

The solution of (20) and (21) is $\theta = \phi \approx 1.146$ which is denoted by θ_0. This means that if both markets co-exist, the Poisson parameter that governs the arrival rates is the same in both markets. Denoting u/v by α, the equilibrium fractions of firms and workers in the two markets satisfy

$$\theta_0 = \frac{\alpha x}{y} = \frac{1 - y}{\alpha (1 - x)}, \qquad (22)$$

and after a few steps we have

$$\begin{aligned} x &= \frac{\theta_0 (\alpha \theta_0 - 1)}{\alpha \left(\theta_0^2 - 1\right)}, & (23) \\ y &= \frac{\alpha \theta_0 - 1}{\theta_0^2 - 1}. & (24) \end{aligned}$$

However, two markets co-exist only if $x \in (0,1)$ and $y \in (0,1)$. These hold only if $\alpha \in \left(\frac{1}{\theta_0}, \theta_0\right)$. We see that two markets co-exist only if there are roughly equally many firms and workers in the economy.

Proposition 1 *i) A mixed-strategy equilibrium is unique, and ii) $x > y$ in the mixed-strategy equilibrium .*

Proof. i) See (23) and (24), ii) Two markets co-exist only if $x \in (0,1)$ and $y \in (0,1)$. These hold only if $\alpha \in \left(\frac{1}{\theta_0}, \theta_0\right)$ .We see that two markets co-exist only if there are roughly equally many firms and workers in the economy. Because $x = \frac{\theta_0}{\alpha} y$ and $\alpha \in \left(\frac{1}{\theta_0}, \theta_0\right)$, we get $\frac{\theta_0}{\alpha} \in (1, 1.313) \Rightarrow x > y$ in equilibrium. ■

A model by Kultti, Miettunen, Takalo and Virrankoski considers buyers and sellers decisions to wait or search, with auction and bargaining as alternative trading mechanisms. It turns out that the model with auction is utillitywise the same as the wage offer model presented here; also the fractions of staying and moving agents are the same as given by formulae (18) and (19) above.

7 Realised Aggregate Distribution

Now we have all the ingredients to determine the realised aggregate wage distribution that emerges in equilibrium where search is two-sided. Using the equilibrium values for x and y gives

$$r(w) = \frac{e^{-\theta_0} (\theta_0 - \alpha)}{(1 - e^{-\theta_0}) (\theta_0 - 1) (1 + \alpha) (1 - w)^2} \approx \frac{3.192 (1.146 - \alpha)}{(1 + \alpha) (1 - w)^2} \text{ if } w \in \left[0, e^{-\theta}\right], \tag{25}$$

$$r(w) = \frac{e^{-\theta_0}}{(1 - e^{-\theta_0}) (\theta_0 - 1) (1 + \alpha)} \left[\frac{\theta_0 - \alpha}{(1 - w)^2} + \frac{\alpha\theta_0 - 1}{w^2}\right] \approx \frac{3.192}{1 + \alpha} \left[\frac{1.146 - \alpha}{(1 - w)^2} + \frac{1.146\alpha - 1}{w^2}\right] \text{ if } w \in \left[e^{-\theta}, 1 - e^{-\phi}\right], \tag{26}$$

$$r(w) = \frac{e^{-\theta_0} (\alpha\theta_0 - 1)}{(1 - e^{-\theta_0}) (\theta_0 - 1) (1 + \alpha) w^2} \tag{27}$$

$$\approx \frac{3.192\,(1.146\alpha - 1)}{(1+\alpha)\,w^2} \text{ if } w \in \left[1 - e^{-\phi}, 1\right]$$

For $u = v$, Figure 1a shows the realised density function if all the firms send offers and all the workers receive them, Figure 1b is the case of all workers sending wage demands; Figure 1c shows the wage density arising from the mixed strategy equilibrium. Figures 2a and 2b show the densities implied by the mixed strategy equilibrium when u/v equals 0.9 or 1.1, respectively.

8 Stability of the Mixed-Strategy Equilibrium

A mixed-strategy equilibrium where $x \in (0,1)$ and $y \in (0,1)$ is economically meaningful only if it is stable. The uniqueness and stability of a mixed-strategy equilibrium is studied in (x, y) -plane (see Figure3). Along firm's (employers') equilibrium EE, we have $e^{-\phi} = 1 - e^{-\theta} - \theta e^{-\theta}$. If workers change their relative participation in the two markets, firms adjust by changing their relative participation such that $e^{-\phi} = 1 - e^{-\theta} - \theta e^{-\theta}$ holds. The respective equilibrium condition for workers, WE, is $e^{-\theta} = 1 - e^{-\phi} - \phi e^{-\phi}$. Parameter $\theta = \frac{xu}{yv}$ governs the arrival of workers' applications to vacancies, whereas workers receive vacancies' offers governed by parameter $\phi = \frac{(1-y)\,v}{(1-x)\,u}$. Denote $\alpha \equiv \frac{u}{v}$.

In order to solve the uniqueness and stability of a mixed-strategy equilibrium we look at the positions of EE and WE. When differentiating EE and WE with respect to x and y, we use the following results: $\frac{\partial \theta}{\partial x} = \frac{\alpha}{y}$, $\frac{\partial \theta}{\partial x} = \frac{-\alpha x}{y^2}$, $\frac{\partial \phi}{\partial x} = \frac{\phi}{1-x}$, and $\frac{\partial \phi}{\partial y} = -\frac{1}{(1-x)\,\alpha}$. Differentiating EE with respect to x and y yields

$$\frac{dy}{dx}\,|_{EE}= \frac{\phi e^{-\phi}\alpha y + \theta e^{-\theta}\alpha^2\,(1-x)}{e^{-\phi}y + \theta^2 e^{-\theta}\alpha\,(1-x)} \tag{28}$$

and along WE,

$$\frac{dy}{dx}\,|_{WE}= \frac{e^{-\theta}\alpha^2\,(1-x) + \phi^2 e^{-\phi}\alpha y}{\theta e^{-\theta}\alpha\,(1-x) + \phi e^{-\phi} y} \tag{29}$$

Both equilibrium curves have a positive slope. Waiting firms fare equally well as moving firms only if an increase in the share of moving workers is accompanied with an increase in the share of waiting firms. The same kind of intuiton applies for workers' equilibrium condition, too.

Next we look whether EE is steeper than WE in equilibrium, or the other way round. Subtracting the right-hand side of 29 from that of 28 yields, after a few steps, that *sign* $\left(\frac{dy}{dx}|_{EE} - \frac{dy}{dx}|_{WE}\right) = sign\left(2\theta\phi - \theta^2\phi^2 - 1\right)$. In equilibrium $\theta = \phi = \theta_0 \approx 1.146$. Function $2x^2 - x^4 - 1$ has a unique maximum of zero at $x = 1$, therefore $2\theta_0^2 - \theta_0^4 - 1 < 0$, which indicates that in equilibrium WE is steeper than EE.

In studying the stability of the mixed-strategy equilibrium, we compare the utility from waiting and moving for firms and workers when they are off the equilibrium curve. The difference of utilities of waiting and moving for firms is $E_r - E_s = 1 - e^{-\theta} - \theta e^{-\theta} - e^{-\phi}$. Suppose that a firm is on EE, and then the fraction of moving workers, x, increases. Then

$$\frac{\partial (E_r - E_s)}{\partial x} = \frac{\partial \left(1 - e^{-\theta} - \theta e^{-\theta} - e^{-\phi}\right)}{\partial x} = \frac{\theta e^{-\theta}\alpha}{y} + \frac{\phi e^{-\phi}}{1-x} > 0, \tag{30}$$

which indicates that for a firm, it is now more profitable to wait than move, and therefore the fraction of waiting firms, y, will increase. For workers, $U_r - U_s = 1 - e^{-\phi} - \phi e^{-\phi} - e^{-\theta}$. If a worker is on WE, and then y increases, the utility difference changes by

$$\frac{\partial (U_r - U_s)}{\partial x} = \frac{\partial \left(1 - e^{-\phi} - \phi e^{-\phi} - e^{-\theta}\right)}{\partial y} = -\frac{\phi e^{-\phi}}{(1-x)\,\alpha} - \frac{e^{-\theta}\alpha x}{y^2} < 0. \tag{31}$$

If the fraction of waiting firms increases, waiting becomes less appealing for workers compared to moving, therefore x will increase. In (x, y) -plane, y decreases above EE and increases below it, and x increase on the left of WE and decreases on the right of it. We thus have

Proposition 2 *The mixed-strategy equilibrium where* $x \in (0,1)$ *and* $y \in (0,1)$ *is stable.*

Endpoints of EE **and** WE.

So far we know that EE and WE are increasing, and at $x \in (0,1)$ and $y \in (0,1)$ they have a unique intersection where $x > y$. We want to find out whether $x = y = 0$ or $x = y = 1$ are also equilibria. If $x = y = 1$, all workers send applications to firms, and none of the firms send offers to workers. However, we know that if such corner equilibria exist, they are necessarily unstable because the equilibrium with $x > 0$ and $y > 0$ is stable.

Firms' equilibrium EE can be written as

$$1-e^{-\dfrac{xu}{yv}}-\frac{xu}{yv}e^{-\dfrac{xu}{yv}}-e^{-\dfrac{(1-y)\,v}{(1-x)\,u}}=0, \tag{32}$$

and workers' equilibrium WE is

$$1-e^{-\dfrac{(1-y)\,v}{(1-x)\,u}}-\frac{(1-y)\,v}{(1-x)\,u}e^{-\dfrac{(1-y)\,v}{(1-x)\,u}}-e^{-\dfrac{xu}{yv}}=0. \tag{33}$$

The behaviour of EE and WE near $(0,0)$ is analyzed first.

1. $(x,y)\rightarrow(0,y)$. i) EE becomes $-e^{-\dfrac{(1-y)\,v}{u}}=0$, which cannot hold for any $y\in[0,1]$. ii) WE becomes $-e^{-\dfrac{(1-y)\,v}{u}}-\dfrac{(1-y)\,v}{u}e^{-\dfrac{(1-y)\,v}{u}}=0$, which cannot hold for any $y\in[0,1]$.

2. $(x,y)\rightarrow(x,0)$. i) EE becomes $1-e^{-\dfrac{v}{(1-x)\,u}}=0$, which cannot hold for any $x\in[0,1]$. ii) WE becomes $1-e^{-\dfrac{v}{(1-x)\,u}}-\dfrac{v}{(1-x)\,u}e^{-\dfrac{v}{(1-x)\,u}}=0$, which does not hold for any $x\in[0,1]$.

Clearly, neither EE nor WE cannot go through $(0,y)$ or $(x,0)$, so they must go through $(0,0)$. We check that this is possible.

Assume that along EE, $x/y\rightarrow a$ as $x\rightarrow0$ and $y\rightarrow0$. Then $\lim\limits_{x\rightarrow0,y\rightarrow0}\dfrac{1-y}{1-x}=\lim\left(\dfrac{1}{1-x}-\dfrac{1}{\dfrac{1}{y}-\dfrac{x}{y}}\right)=1-\lim\dfrac{1}{\dfrac{1}{y}-a}=1$. Then $\lim\limits_{x\rightarrow0,y\rightarrow0}\left(1-e^{-\theta}-\theta e^{-\theta}-e^{-\phi}\right)=1-e^{-a\alpha}-a\alpha e^{-a\alpha}-e^{-1/\alpha}$, which, by for example letting $\alpha=1$, equals zero if $a=1.285$. For any other $\alpha>0$ one can find $a>0$ that satisfies EE going through $(0,0)$.

Assume that along WE, $x/y\rightarrow b$ as $x\rightarrow0$ and $y\rightarrow0$. Then $\lim\limits_{x\rightarrow0,y\rightarrow0}\left(1-e^{-\phi}-\phi e^{-\phi}-e^{-\theta}\right)=1-e^{-1/\alpha}-\dfrac{1}{\alpha}e^{-1/\alpha}-e^{-b\alpha}$, which equals zero if $\alpha=1$ and $b=1.33$. Because $a<b$, EE is above WE near $(0,0)$, which is does not contradict WE being steeper than EE in their intersection at strictly positive x and y.

Next check the curves' positions near $(1,1)$.

1. $(x, y) \to (x, 1)$. i) EE becomes $-e^{-\frac{xu}{v}} - \frac{xu}{v} e^{-\frac{xu}{v}} = 0$, which cannot hold for any $x \in [0, 1]$. ii) WE becomes $-e^{-\frac{xu}{v}} = 0$, which cannot hold for any $x \in [0, 1]$.

2. $(x, y) \to (1, y)$. i) EE becomes $1 - e^{-\frac{u}{yv}} - \frac{u}{yv} e^{-\frac{u}{yv}} = 0$, which cannot hold for any $y \in [0, 1]$. ii) WE becomes $1 - e^{-\frac{u}{yv}} = 0$, which cannot hold for any $y \in [0, 1]$.

We see that EE and WE must go through $(1, 1)$. As x and y approach 1, assume that $\frac{1-y}{1-x} \to c$ along EE and $\frac{1-y}{1-x} \to g$ along WE. Then EE becomes $\lim_{x \to 1, y \to 1} \left(1 - e^{-\theta} - \theta e^{-\theta} - e^{-\phi}\right) = 1 - e^{-\alpha} - \alpha e^{-\alpha} - e^{-c/\alpha} = 0$, which holds for example if $\alpha = 1$ and $c = 1.33$. In the limit WE equals $\lim_{x \to 1, y \to 1} \left(1 - e^{-\phi} - \phi e^{-\phi} - e^{-\theta}\right) = 1 - e^{-g/\alpha} - \frac{g}{\alpha} e^{-g/\alpha} - e^{-\alpha} = 0$; with $\alpha = 1$ it holds if $g = 1.285$. Near $(1, 1)$, WE lies above EE, which is consistent with WE being steeper than EE in their intersection at strictly positive x and y. We can conclude the following:

Proposition 3 *Equilibrium curves EE and WE go through $(0, 0)$ and $(1, 1)$.*

That is, $(0, 0)$ and $(1, 1)$ are genuine mixed-strategy equilibria. However, because the equilibrium with strictly positive x and y is stable, the corner equilibria are unstable. The urn-ball literature has thus far concentrated on these unstable equilibria.

9 One-sided Search

Two markets co-exist only if $x \in (0, 1)$ and $y \in (0, 1)$. These hold only if $\alpha \in \left(\frac{1}{\theta_0}, \theta_0\right)$. Outside this range, search is one-sided. We can directly use a result derived in Kultti, Miettunen, Takalo, and Virrankoski (2004):

Proposition 4 *i) If $\alpha < \frac{1}{\theta_0}$, then $x = y = 0$, ii) If $\alpha > \theta_0$, then $x = y = 1$.*

Proof. The proof is lengthy and is presented in Kultti, Miettunen, Takalo, and Virrankoski (2004). ■

If u/v is small, all the firms send wage offers and none of the workers send wage demands. If u/v is large, all the workers send wage demands and none of the firms send wage offers. The idea of the proof is the following: Assume that $\alpha > \theta_0$, and all the firms send wage offers and none of the workers send wage demands. It can be shown that there exists a deviating coalition of firms and workers such that all its members would be better off in a market where workers send wage demands and none of the firms sends offers. Then, the original market cannot be an equilibrium. On the other hand, if $\alpha > \theta_0$ and all the workers send wage demands and none of the firms sends offers, a deviating coalition does not exist. Workers would prefer the new market where firms send offers only if the Poisson parameter in the new market is large enough, whereas firms prefer the new market only if the Poisson parameter is small enough. It can be shown that if $\alpha > \theta_0$, the required regions for the Poisson parameter do not overlap, thus a deviating coalition cannot exist. If $\alpha < \dfrac{1}{\theta_0}$, an analogous reasoning applies.

If there are a lot of unemployed compared to vacancies, Proposition 4 implies that the wage offer density function and the density function for realised wages are decreasing, whereas in case of relatively numerous vacancies, the density functions are increasing.

10 Conclusion

We derive a wage density function for homogenous firms and homogenous workers. Both firms and workers can send wage demands or offers, which is what we often see happening in real labour markets. We derive a mixed strategy for offers and demands and solve the equilibrium fractions of firms and workers who are engaged in sending or receiving offers. For small wages the density function is low and increasing, for high wages it is low and decreasing, and for middle-range wages it is high and u-shaped. The wage distribution the model produces is not exactly the one observed empirically, but it is fairly close to that. It is notable that we get this hump-shaped distribution without assuming any kind of heterogeneity among workers or firms. The equilibrium associated with the hump-shaped wage distribution is evolutionarily stable, whereas the equilibria that are associated with monotonous distributions are unstable. Another interesting result is that the mixed strategies that employers and job seekers use are utilitywise equivalent to auctions where the agents know the number of their competitors. The matching function used is well determined with a firm microfoundation. In this model one can do rigorous comparative statics as nothing comes outside of the model. In particular, one can determine the response of duration of unemployment spells when the measure of workers or firms changes, or when the expected life-span of matches change (we do that in another article). The model is well suited also to to consider the implications of worker/firm heterogeneity on wage distribution. In the next version of the paper we will continue the analysis of the dynamic model 2

11 Appendix 1: Dynamic Model 1

Case 1: Firms send offers to workers

Let us study a situation where the workers are like urns and employers as balls. Our aim is to determine the mixed strategy of the employers in a dynamic model focusing on a steady-state. It turns out that the agents' expected utilities are the same as in a corresponding model where the wages are determined by auction. We need the results of the auction to determine the mixed strategies, and that in mind we first determine the agents' expected utilities under auction. The workers' and the employers' utilities are determined by the following equations

$$V_w = (e^{-\phi} + \phi e^{-\phi})\delta V_w + (1 - e^{-\phi} - \phi e^{-\phi})(1 - \delta V_e), \tag{34}$$

$$V_e = e^{-\phi}(1 - \delta V_w) + (1 - e^{-\phi})\delta V_{e.} \tag{35}$$

In 34, $e^{-\phi}$ is the probability that no firm comes to the worker, and $\phi e^{-\phi}$ stands for the probability of just one firm arriving, in which case the firm makes a take-it-or-leave-it offer. In both these cases, the worker continues to the next period with his discounted reservation value δV_w. If he gets two or more firms, the firms engage in Bertrand competition for the right to employ the worker. The firms, regerdless of which of them employs the worker, get their discounted reservation value δV_e, and the worker gets $1-\delta V_e$. In 35, with probability $e^{-\phi}$ the firm is the only one that meets the worker, the firm makes take-it-or-leave-it offer and gets one minus the worker's discounted reservation value. If the firms has at least one competitor, it gets it discounted reservation value.

From these one gets explicit expressions

$$V_w = \frac{1 - e^{-\phi} - \phi e^{-\phi}}{1 - \delta\phi e^{-\phi}}, \tag{36}$$

$$V_e = \frac{e^{-\phi}}{1 - \delta\phi e^{-\phi}}. \tag{37}$$

Let us now leave the auction and assume that employers use a continuous mixed strategy H with support $[a, b]$. An employer's expected utility when he offers wage $w \in [a, b]$ is given by

$$V_e(w) = \sum_{k=0}^{\infty} e^{-\phi} \frac{\phi^k}{k!} \left[H^k(w)(1 - w) + \left(1 - H^k(w)\right) \delta V_e \right], \tag{38}$$

which after some simplification equals

$$V_e(w) = (1 - w) e^{-\phi\left(1 - H^k(w)\right)} + \delta V_e \left(1 - e^{-\phi\left(1 - H^k(w)\right)}\right). \tag{39}$$

Next we use the fact that any wage in the support of the mixed strategy yields the same utility to the employer, in particular, this holds for the lowest and the highest wages

$$V_e = V_e(a) = (1 - a)e^{-\phi} + \delta V_e (1 - e^{-\phi}) = V_e(b) = 1 - b. \tag{40}$$

From this we can solve for

$$b = 1 - (1 - a)e^{-\phi} - \delta V_e (1 - e^{-\phi}) = 1 - e^{-\phi} + \delta V_w e^{-\phi} - \delta V_e (1 - e^{-\phi}), \tag{41}$$

where the last equality is based on the fact that the lowest wage in the support of the mixed strategy must equal the workers' outside option, i.e. it must make them indifferent between accepting it and continuing search. Thus, we have that

$$a = \delta V_w. \tag{42}$$

We let $h(w) = H'(w)$ and determine the workers' expected utility

$$V_w = \sum_{k=1}^{\infty} e^{-\phi} \frac{\phi^k}{k!} \int_a^b wkh(w) H^{k-1}(w) dw + e^{-\phi} \delta V_w \tag{43}$$

which equals

$$V_w = \sum_{k=1}^{\infty} e^{-\phi} \frac{\phi^k}{k!} \left[\Big/_a^b H^k(w) w - \int_a^b H^k(w) dw \right] = \sum_{k=1}^{\infty} e^{-\phi} \frac{\phi^k}{k!} \left[b - \int_a^b H^k(w) dw \right] \tag{44}$$

which in turn equals, using Fubini's theorem,

$$V_w = b - \int_a^b e^{-\phi(1-H(w))} dw. \tag{45}$$

Inserting this and (40) into (41) yields

$$b = 1 - e^{-\phi} - \frac{\delta}{1-\delta} e^{-\phi} \int_a^b e^{-\phi(1-H(w))} dw. \tag{46}$$

Next we impose that the expected utility of an employer equals that of an employer with auction

$$V_e = V_e(b) = 1 - b = \frac{e^{-\phi}}{1 - \delta\phi e^{-\phi}} \tag{47}$$

which yields the following formula:

$$b = \frac{1 - e^{-\phi} - \delta\phi e^{-\phi}}{1 - \delta\phi e^{-\phi}}. \tag{48}$$

From (40) we get by similarly

$$V_e = V_e(a) = \left(1 - \delta + \delta e^{-\phi}\right)^{-1} (1-a) e^{-\phi} = \frac{e^{-\phi}}{1 - \delta\phi e^{-\phi}}. \tag{49}$$

From this we can solve

$$a = \delta V_w = \delta \frac{1 - e^{-\phi} - \delta\phi e^{-\phi}}{1 - \delta\phi e^{-\phi}} = \delta b. \tag{50}$$

Using the fact that $V_e(w) = V_e(b)$, solving $V_e(w)$ from (39) and equating it with (47) yields

$$e^{-\phi(1-H(w))} \left[1 - w - \delta \frac{e^{-\phi}}{1 - \delta\phi e^{-\phi}}\right] = (1-\delta) \frac{e^{-\phi}}{1 - \delta\phi e^{-\phi}}. \tag{51}$$

From this we can solve the equilibrium mixed strategy

$$H(w) = 1 - \frac{1}{\phi} \ln\left(1 - w - \delta \frac{e^{-\phi}}{1 - \delta\phi e^{-\phi}}\right) + \frac{1}{\phi} \ln\left((1-\delta) \frac{e^{-\phi}}{1 - \delta\phi e^{-\phi}}\right). \tag{52}$$

The equilibrium mixed strategy is unobservable while the realised wages that result from it generate an observable wage distribution. We denote the cumulative distribution function for realised wages by M and the corresponding density function is denoted by m. Let us determine the probability that wage w is observed.

$$\left(1 - e^{-\phi}\right) m(w) = \sum_{k=1}^{\infty} e^{-\phi} \frac{\phi^k}{k!} k h(w) H^{k-1}(w) = \phi e^{-\phi(1-H(w))} h(w) \tag{53}$$

From this we get

$$m(w) = \frac{e^{-\phi(1-H(w))} - e^{-\phi}}{1 - e^{-\phi}} \tag{54}$$

Inserting (52) above and manipulating a little yields an explicit formula

$$m(w) = \frac{(1-\delta)e^{-\phi}(1-\delta\phi e^{-\phi})}{(1-e^{-\phi})\left[(1-w)(1-\delta\phi e^{-\phi}) - \delta e^{-\phi}\right]^2}. \tag{55}$$

Case 2: Workers send applications to firms

In the standard auction model the utilities of firms and workers are

$$V_e = (e^{-\theta} + \theta e^{-\theta})\delta V_e + (1 - e^{-\theta} - \theta e^{-\theta})(1 - \delta V_w), \tag{56}$$

$$V_w = e^{-\theta}(1 - \delta V_w) + (1 - e^{-\theta})\delta V_w. \tag{57}$$

Solving these yields

$$V_e = \frac{1 - e^{-\theta} - \theta e^{-\theta}}{1 - \delta\theta e^{-\theta}}, \tag{58}$$

$$V_w = \frac{e^{-\theta}}{1 - \delta\theta e^{-\theta}}. \tag{59}$$

Next assume that workers use a continuous mixed strategy F with support $[A, B]$ .A worker's expected utility when he asks wages $w \in [A, B]$ is

$$V_w(w) = \sum_{k=0}^{\infty} e^{-\theta}\frac{\theta^k}{k!}\left([1 - F(w)]^k w + \left(1 - [1 - F(w)]^k\right)\delta V_w\right) \tag{60}$$

$$\Leftrightarrow \; V_w(w) = we^{-\theta F(w)} + \left(1 - e^{-\theta F(w)}\right)\delta V_w \tag{61}$$

$$\Leftrightarrow \; V_w = \frac{we^{-\theta F(w)}}{1 - \delta + \delta e^{-\theta F(w)}}. \tag{62}$$

Any wage in support $[A, B]$ yields the same utility to a worker, especially $V_w(A) = V_w(B)$:

$$V_w(A) = Ae^{-\theta F(A)} + \left(1 - e^{-\theta F(A)}\right)\delta V_w = Be^{-\theta F(B)} + \left(1 - e^{-\theta F(A)}\right)\delta V_w = V_w(B), \tag{63}$$

and using $F(A) = 0$ and $F(B) = 1$ we have

$$V_w = A, \tag{64}$$

$$A = Be^{-\theta} + \left(1 - e^{-\theta}\right). \tag{65}$$

The highest offer the worker makes must leave the firm its reservation value:

$$B = 1 - \delta V_e. \tag{66}$$

and A can be written as

$$A = (1 - \delta V_e) + \left(1 - e^{-\theta}\right) \delta V_w. \tag{67}$$

Let $f(w) \equiv F'(w)$ and determine a firm's expected utility as

$$V_e = \sum_{k=1}^{\infty} \frac{e^{-\theta}\theta^k}{k!} \int_A^B (1-w) f(w) k \left[1 - F(w)\right]^{k-1} dw + e^{-\theta}\delta V_e \tag{68}$$

$$\begin{aligned}
&= \sum_{k=1}^{\infty} \frac{e^{-\theta}\theta^k}{k!} \left(- \left[1 - F(B)\right]^k + \left[1 - F(A)\right]^k \right) \\
&\quad - \int_A^B \sum_{k=1}^{\infty} \frac{e^{-\theta}\theta^k}{k!} f(w) k \left[1 - F(w)\right]^{k-1} w dw + e^{-\theta}\delta V_e \\
&= \sum_{k=1}^{\infty} \frac{e^{-\theta}\theta^k}{k!} - \int_A^B \sum_{k=1}^{\infty} \frac{e^{-\theta}\theta^k}{(k-1)!} f(w) \left[1 - F(w)\right]^{k-1} w dw + e^{-\theta}\delta V_e \\
&= 1 - e^{-\theta} - \theta \int_A^B f(w) w \sum_{k=0}^{\infty} \frac{e^{-\theta}\theta^k \left[1 - F(w)\right]^k}{k!} dw + e^{-\theta}\delta V_e \\
&= 1 - e^{-\theta} - \theta \int_A^B e^{-\theta F(w)} f(w) w dw + e^{-\theta}\delta V_e \\
&= 1 - e^{-\theta} + B e^{-\theta F(B)} - A e^{-\theta F(A)} - \int_A^B e^{-\theta F(w)} dw + e^{-\theta}\delta V_e \\
&= 1 - e^{-\theta} + B e^{-\theta} - A - \int_A^B e^{-\theta F(w)} dw + e^{-\theta}\delta V_e.
\end{aligned}$$

Then subsitute $1 - \delta V_e$ for B and rearrange to have

$$V_e = 1 - A - \int_A^B e^{-\theta F(w)} dw. \tag{69}$$

Using (68) and $A = V_w$ in (66) we get

$$A = \left[1 - \delta \left(1 - A - \int_A^B e^{-\theta F(w)} dw\right)\right] e^{-\theta} + \delta \left(1 - e^{-\theta}\right) A \tag{70}$$

$$\Leftrightarrow$$

$$A = \frac{1 - \delta e^{-\theta} + \delta e^{-\theta} \int_A^B e^{-\theta F(w)} dw}{1 - \delta}. \tag{71}$$

The lower bound of the support of wage asks cannot be determined explicitly.

Next we impose that the expected utility of a worker equals that of a worker with auction:

$$V_w = \frac{e^{-\theta}}{1 - \delta\theta e^{-\theta}} = A. \tag{72}$$

Utilizing (62) yields

$$V_w = Be^{-\theta} + \delta V_w \left(1 - e^{-\theta}\right), \tag{73}$$

which gives, along with (72), that

$$V_w = \frac{Be^{-\theta}}{1 - \delta + \delta e^{-\theta}} = \frac{e^{-\theta}}{1 - \delta\theta e^{-\theta}}. \tag{74}$$

Solving for B gives

$$B = \frac{1 - \delta + \delta e^{-\theta}}{1 - \delta\theta e^{-\theta}}. \tag{75}$$

Equating V_w given by (62) and that given by (74) yields

$$\frac{we^{-\theta F(w)}}{1 - \delta + \delta e^{-\theta F(w)}} = \frac{e^{-\theta}}{1 - \delta\theta e^{-\theta}} \tag{76}$$

$$\Leftrightarrow$$

$$e^{-\theta F(w)} = \frac{\dfrac{(1 - \delta)\, e^{-\theta}}{1 - \delta\theta e^{-\theta}}}{w - \dfrac{\delta e^{-\theta}}{1 - \delta\theta e^{-\theta}}} \tag{77}$$

Taking logarithms and arranging results in

$$F(w) = \frac{1}{\theta}\left[\ln\left(w - \frac{\delta e^{-\theta}}{1 - \delta\theta e^{-\theta}}\right) - \ln\left(\frac{(1 - \delta)\, e^{-\theta}}{1 - \delta\theta e^{-\theta}}\right)\right]. \tag{78}$$

We denote again the cumulative distribution function for realised wages by $G(w)$ and the corresponding density function is denoted by $g(w)$.

$$G(w) = \frac{1 - e^{-\theta F(w)}}{1 - e^{-\theta}} \tag{79}$$

$$= \frac{1}{1 - e^{-\theta}} - \frac{(1 - \delta)e^{-\theta}}{\left(1 - e^{-\theta}\right)\left[w(1 - \delta\theta e^{-\theta}) - \delta e^{-\theta}\right]}. \tag{80}$$

The density function is

$$g(w) = \frac{\left(1 - \delta\theta e^{-\theta}\right)(1 - \delta)e^{-\theta}}{\left(1 - e^{-\theta}\right)\left[w(1 - \delta\theta e^{-\theta}) - \delta e^{-\theta}\right]^2}. \tag{81}$$

11.1 Equilibrium of Dynamic Model 1

In equilibrium, workers are indifferent between sending applications and receiving offers from firms, and firms are indifferent between making offers and receiving applications from workers. The equilibrium condition for a firm, EE, is

$$\frac{e^{-\phi}}{1-\delta\phi e^{-\phi}} = \frac{1-e^{-\theta}-\theta e^{-\theta}}{1-\delta\theta e^{-\theta}}, \tag{82}$$

and the respective condition WE for a worker is

$$\frac{e^{-\theta}}{1-\delta\theta e^{-\theta}} = \frac{1-e^{-\phi}-\phi e^{-\phi}}{1-\delta\phi e^{-\phi}}. \tag{83}$$

Proceeding as in the static case, it turns out that

$$x = \frac{\theta_0\left(\alpha\theta_0-1\right)}{\alpha\left(\theta_0^2-1\right)}, \tag{84}$$

$$y = \frac{\alpha\theta_0-1}{\theta_0^2-1}. \tag{85}$$

as in the static model. Along WE,

$$\frac{dy}{dx}\mid_{WE}= \frac{C\frac{d\phi}{dx}-A\frac{d\theta}{dx}}{A\frac{d\theta}{dy}-C\frac{d\phi}{dy}}, \tag{86}$$

where $A=\frac{-e^{-\theta}}{\left(1-\delta\theta e^{-\theta}\right)^2}$ and $C=\frac{e^{-\phi}\left[(1-\delta)\phi+\delta\left(1-e^{-\phi}\right)\right]}{\left(1-\delta\phi e^{-\phi}\right)^2}$. Along EE,

$$\frac{dy}{dx}\mid_{EE}= \frac{B\frac{d\theta}{dx}-D\frac{d\phi}{dx}}{D\frac{d\phi}{dy}-B\frac{d\theta}{dy}}, \tag{87}$$

where $B=\frac{e^{-\theta}\left[(1-\delta)\theta+\delta\left(1-e^{-\theta}\right)\right]}{\left(1-\delta\theta e^{-\theta}\right)^2}$ and $D=\frac{-e^{-\phi}}{\left(1-\delta\phi e^{-\phi}\right)^2}$. Curve WE is steeper than EE if $(AD-BC)\left(\frac{d\phi}{dx}\frac{d\theta}{dy}-\frac{d\theta}{dx}\frac{d\phi}{dy}\right)>0$. In equilibrium $\phi=\theta=\theta_0\approx 1.146$, and the sign of $AD-BC$ turns out to be equal to the sign of $(1-\delta)\left(1-\theta_0\right)$, which is negative. Expression $\frac{d\phi}{dx}\frac{d\theta}{dy}-\frac{d\theta}{dx}\frac{d\phi}{dy}$ simplifies to $\frac{y-x}{(1-x)^2 y^2}$, which is negative by $x>y$ in equilibrium. These results yield

Proposition 5 *In dynamic model 1, the mixed-strategy equilibrium where* $x\in(0,1)$ *and* $y\in(0,1)$ *is stable.*

As in the static case, the endpoints of both EE and WE are at $(0,0)$ and at $(1,1)$.

11.2 Distribution of Realized Wages in the Dynamic Model 1

Calculating the distribution of realised wages goes analogously to the corresponding task in the static model:

$$r(w) = \begin{cases} \dfrac{m(w)(1-x)\alpha}{(1-x)\alpha + y} & \text{when } a \le w \le A \\ \dfrac{m(1-x)\alpha + g(w)y}{(1-x)\alpha + y} & \text{when } \; A < w \le b \\ \dfrac{g(w)y\alpha}{(1-x)\alpha + y} & \text{when } \; b < w \le B \end{cases} \tag{88}$$

where $a = \delta \dfrac{1 - e^{-\phi} - \delta\phi e^{-\phi}}{1 - \delta\phi e^{-\phi}}$, $b = a/\delta$, $A = \dfrac{e^{-\theta}}{1 - \delta\theta e^{-\theta}}$ and $B = \dfrac{1 - \delta + \delta e^{-\theta}}{1 - \delta\theta e^{-\theta}}$.

12 Appendix 2: Dynamic Model 2

Case 1: Firms send offers to workers

Assume that the total number of workers is W and the total number of employers is E. Some of them are matched with each other in productive activities, while others are looking for a partner. The number of unemployed is denoted by U and the number of vacancies by V. Procuction happens in pairs, therefore

$$W - U = E - V. \tag{89}$$

For the moment we focus just on the matching market which is assumed to be in a steady state. The only complication to the standard set-up is an exogenous separation probability b, and the fact that a worker who is employed at wage w gets the wage each period as long as the employment relationship lasts, and correspondingly the employer get $1 - w$ each period.

Auction

Let the unemployed be urns and the vacancies balls, and let $\phi = \frac{V}{U}$. First we determine their expected life time utilities when wages are determined in auction. The timing is as follows: We determine the expected life time utility of an unemployer worker and a vacancy at the very beginning of a period. The utility of a worker or an employer that has a partner is evaluated right after that, i.e. within the same period before anything else happens. After that the parties produce and get their shares of the production. After that separations take place. The utility of an unemployed worker is determined by

$$A_u = \left(e^{-\phi} + \phi e^{-\phi}\right) \delta A_u + \left(1 - e^{-\phi} - \phi e^{-\phi}\right) A_u(\overline{w}) \tag{90}$$

where $\overline{w}$ is the wage that vacancies offer when there are two or more vacancies competing for a worker. We take it as given for now, and determine the equilibrium value later on. We have also used the fact that when a worker meets exactly one vacancy the vacancy makes a take-it-or-leave-it offer that leaves no surplus to the worker. The utility of a matched worker with wage $\overline{w}$ is determined by

$$A_u(\overline{w}) = \overline{w} + \delta b A_u + \delta(1-b)A_u(\overline{w}). \tag{91}$$

The expected utility of a vacancy is determined by

$$A_v = e^{-\phi} A_v \left(1 - \underline{w}\right) + \left(1 - e^{-\phi}\right) \delta A_v \tag{92}$$

where $\underline{w}$ is the wage that a vacancy offers when it gets to make a take-it-or-leave-it offer. The expected utility of an employer who employs at wage $\underline{w}$ is determined by

$$A_v(1-\underline{w}) = 1 - \underline{w} + \delta b A_v + \delta(1-b)A_v(1-\underline{w}). \tag{93}$$

From these equations we can determine the expected utilities as the function of the wages

$$A_u = \frac{\left(1 - e^{-\phi} - \phi e^{-\phi}\right) \overline{w}}{(1-\delta)\left(1 - \delta(1-b)\left(e^{-\phi} + \phi e^{-\phi}\right)\right)} \tag{94}$$

$$A_u(\overline{w}) = \frac{\left(1 - \delta(e^{-\phi} + \phi e^{-\phi})\right) \overline{w}}{(1-\delta)\left(1 - \delta(1-b)\left(e^{-\phi} + \phi e^{-\phi}\right)\right)} \tag{95}$$

$$A_v = \frac{e^{-\phi}(1-\underline{w})}{(1-\delta)\left(1 - \delta(1-b)\left(1 - e^{-\phi}\right)\right)} \tag{96}$$

$$A_v(1-\underline{w}) = \frac{\left(1 - \delta + \delta e^{-\phi}\right)(1-\underline{w})}{(1-\delta)\left(1 - \delta(1-b)\left(1 - e^{-\phi}\right)\right)} \tag{97}$$

Next we determine the two possible equilibrium wages. The higher wage $\overline{w}$, that comes about when several vacancies compete for a worker, must be such that all the vacancies are indifferent between paying the wage and continuing search for a worker, i.e.

$$1 - \overline{w} + \delta b A_v + \delta(1-b)A_v(1-\overline{w}) = \delta A_v. \tag{98}$$

Similarly, the lower wage $\underline{w}$, that comes about when a vacancy meets an unemployed alone and gets to make a take-it-or-leave-it offer, is such that the unemployed is indifferent between accepting the wage and continuing search, i.e.

$$\underline{w} + \delta b A_u + \delta(1-b)A_u(\underline{w}) = \delta A_u. \tag{99}$$

Using (94)-(97), and replacing $\overline{w}$ by $\underline{w}$ in (102), and $\underline{w}$ by $\overline{w}$ in (93), we can solve

$$\overline{w} = \frac{1-\delta\left(1-b\right)\left(e^{-\phi}+\phi e^{-\phi}\right)}{1-\delta(1-b)\phi e^{-\phi}} \tag{100}$$

$$\underline{w} = \frac{\delta(1-b)\left(1-e^{-\phi}-\phi e^{-\phi}\right)}{1-\delta(1-b)\phi e^{-\phi}} \tag{101}$$

Using these data we can finally solve for the expected utility of an unemployed worker who waits

$$A_u = \frac{1-e^{-\phi}-\phi e^{-\phi}}{\left(1-\delta\right)\left(1-\delta(1-b)\phi e^{-\phi}\right)} \tag{102}$$

and for the expected utility of a vacancy that moves

$$A_v = \frac{e^{-\phi}}{(1-\delta)(1-\delta(1-b)\phi e^{-\phi})} \tag{103}$$

Mixed strategy

The expected utility of an unemployed when the vacacies use a mixed strategy $H(w)$ with support $[l,h]$ is determined by

$$M_u = e^{-\phi}\delta M_u + \sum_{k=1}^{\infty} e^{-\phi}\frac{\phi^k}{k!}\int_l^h kh(w)H^{k-1}(w)M_u(w)dw. \tag{104}$$

Similarly, the utility of a worker who is employed at wage w is given by

$$M_u(w) = w + \delta b M_u + \delta(1-b)M_u(w). \tag{105}$$

Solving $M_u(w)$ and inserting it back to (104) yields the following formula where the last two terms result from partial integration

$$M_u = e^{-\phi}\delta M_u + \sum_{k=1}^{\infty} e^{-\phi}\frac{\phi^k}{k!}\int_l^h H^k(w)(1-\delta+\delta b)^{-1}\delta b M_u + \tag{106}$$

$$\sum_{k=1}^{\infty} e^{-\phi}\frac{\phi^k}{k!}\int_l^h H^k(w)(1-\delta+\delta b)^{-1}w -$$

$$\sum_{k=1}^{\infty} e^{-\phi}\frac{\phi^k}{k!}\int_l^h H^k(w)(1-\delta+\delta b)^{-1}dw.$$

Finally, we can simplify this by doing the summations and by changing the order of the summation and integration in the last sum

$$M_u = (1-\delta)^{-1}(1-\delta(1-b)e^{-\phi})^{-1}\left\{h - le^{-\phi} - \int_l^h e^{-\phi(1-H(w))}dw\right\}. \tag{107}$$

The expected utility of a vacancy must be the same regardless of which element it chooses from the support of its mixed strategy. Let us determine the utility of a vacancy that uses l. It is determined by

$$M_v^{1-l} = e^{-\phi}\left[1 - l + \delta b M_v^{1-l} + \delta(1-b)M_v(1-l)\right] + (1-e^{-\phi})\delta M_v^{1-l}. \tag{108}$$

Analogously, if a vacancy uses h its utility is determined by

$$M_v^{1-h} = 1 - h + \delta b M_v^{1-h} + \delta(1-b)M_v(1-h) \tag{109}$$

as offering the highest possible wage means that the wage is always accepted. Finally, if the vacancy offers wage w its utility is determined by

$$M_v^{1-w} = \sum_{k=0}^{\infty} e^{-\phi}\frac{\phi^k}{k!}\left[\begin{array}{c} H^k(w)\left(1 - w + \delta b M_v^{1-w} + \delta(1-b)M_v(1-w)\right) \\ +\left(1 - H^k(w)\right)\delta M_v^{1-w}\end{array}\right] \tag{110}$$

The utilities of being in an employment relationship at a specific wage are easily determined from equations

$$M_v(1-l) = 1 - l + \delta b M_v^{1-l} + \delta(1-b)M_v(1-l) \tag{111}$$

$$M_v(1-h) = 1 - h + \delta b M_v^{1-h} + \delta(1-b)M_v(1-h) \tag{112}$$

$$M_v(1-w) = 1 - w + \delta b M_v^{1-w} + \delta(1-b)M_v(1-w) \tag{113}$$

Solving from these the expected utility and inserting in formulae (112)-(110) and in turn forcing them to yield the same expected utility as auction, namely that given by (96) allows us to solve for the endpoints of the support of the mixed strategy as well as the mixed strategy itself

$$l = \frac{\delta(1-b)\left(1 - e^{-\phi} - \phi e^{-\phi}\right)}{1 - \delta(1-b)\phi e^{-\phi}} \tag{114}$$

$$h = \frac{1 - \delta(1-b)\phi e^{-\phi} - e^{-\phi}}{1 - \delta(1-b)\phi e^{-\phi}} \tag{115}$$

$$H(w) = \frac{1}{\phi} \ln \frac{1 - \delta(1-b)}{(1-w)\left[1 - \delta(1-b)\phi e^{-\phi}\right] - \delta(1-b)e^{-\phi}} \tag{116}$$

We denote the cumulative distribution function for realised wages by $M(w)$ and the corresponding density function is denoted by $m(w)$.

$$\begin{aligned} M(w) &= \frac{e^{-\phi(1-H(w))} - e^{-\phi}}{1 - e^{-\phi}} && (117) \\ &= \frac{(1-\delta(1-b))e^{-\phi}}{(1-e^{-\phi})\left[(1-w)(1-\delta(1-b)\phi e^{-\phi}) - \delta(1-b)e^{-\phi}\right]} - \frac{e^{-\phi}}{1-e^{-\phi}}. && (118) \end{aligned}$$

The density function is

$$m(w) = \frac{\left(1 - \delta(1-b)\phi e^{-\phi}\right)(1-\delta(1-b))e^{-\phi}}{(1-e^{-\phi})\left[(1-w)(1-\delta(1-b)\phi e^{-\phi}) - \delta(1-b)e^{-\phi}\right]^2} \tag{119}$$

Case 2: Workers send applications to firms

Auction

Let $\theta = \frac{u}{v}$. The utility of a vacancy is determined by

$$A_v = \left(e^{-\theta} + \theta e^{-\theta}\right)\delta A_v + \left(1 - e^{-\theta} - \theta e^{-\theta}\right) A_v(1 - \underline{w}) \tag{120}$$

where $\underline{w}$ is the wage that a worker offers when there are two or more workers competing for a vacancy. Note that when a vacancy meets exactly one worker the worker makes a take-it-or-leave-it offer that leaves no surplus to the vacancy. The utility of an employer who employes at wage $\underline{w}$

$$A_v(1-\underline{w}) = 1 - \underline{w} + \delta b A_v + \delta(1-b)A_v(1-\underline{w}). \tag{121}$$

The expected utility of an unemployed worker is determined by

$$A_u = e^{-\theta} A_u(\overline{w}) + \left(1 - e^{-\theta}\right)\delta A_u \tag{122}$$

where $\overline{w}$ is the wage that a worker offers when it gets to make a take-it-or-leave-it offer. The utility of a matched worker who is paid wage $\overline{w}$

$$A_u(\overline{w}) = \overline{w} + \delta b A_u + \delta(1-b)A_u(\overline{w}). \tag{123}$$

From (120)-(123) we can solve

$$A_v = \frac{\left(1 - e^{-\theta} - \theta e^{-\theta}\right)(1 - \underline{w})}{(1-\delta)\left(1 - \delta(1-b)\left(e^{-\theta} + \theta e^{-\theta}\right)\right)} \tag{124}$$

$$A_v\left(1 - \underline{w}\right) = \frac{\left(1 - \delta(e^{-\theta} + \theta e^{-\theta})\right)(1 - \underline{w})}{(1-\delta)\left(1 - \delta(1-b)\left(e^{-\theta} + \theta e^{-\theta}\right)\right)} \tag{125}$$

$$A_u = \frac{e^{-\theta}\overline{w}}{(1-\delta)\left(1 - \delta(1-b)\left(1 - e^{-\theta}\right)\right)} \tag{126}$$

$$A_u\left(\overline{w}\right) = \frac{\left(1 - \delta + \delta e^{-\theta}\right)(1 - \overline{w})}{(1-\delta)\left(1 - \delta(1-b)\left(1 - e^{-\theta}\right)\right)} \tag{127}$$

The two possible equilibrium wages are :

$\underline{w}$: Several workers compete for a vacancy, all unemployed are are indifferent between working with the wage and continuing search

$$\underline{w} + \delta b A_u + \delta\left(1 - b\right) A_u(\underline{w}) = \delta A_u. \tag{128}$$

$\overline{w}$: A worker is the only applicant and gets to make a take-or-leave-it offer. The firm is indifferent between accepting the wage and continuing search

$$1 - \overline{w} + \delta b A_v + \delta\left(1 - b\right) A_v(1 - \overline{w}) = \delta A_{v.} \tag{129}$$

Using (124)-(127), and replacing $\underline{w}$ by $\overline{w}$ in (121), and $\overline{w}$ by $\underline{w}$ in (123) we can solve

$$\overline{w} = \frac{1 - \delta\left(1 - b\right)\left(1 - e^{-\theta}\right)}{1 - \delta(1-b)\theta e^{-\theta}}, \tag{130}$$

$$\underline{w} = \frac{\delta\left(1 - b\right)e^{-\theta}}{1 - \delta(1-b)\theta e^{-\theta}}. \tag{131}$$

Using (130) and (131) we can solve for the expected utility of a vacancy that waits

$$A_v = \frac{1 - e^{-\theta} - \theta e^{-\theta}}{(1-\delta)\left(1 - \delta(1-b)\theta e^{-\theta}\right)}. \tag{132}$$

and for the expected utility of a moving unemployed worker

$$A_u = \frac{e^{-\theta}}{(1-\delta)(1 - \delta(1-b)\theta e^{-\theta})}. \tag{133}$$

Mixed strategy

The expected utility of a vacancy when the workers use a mixed strategy $F(w)$ with support $[l, h]$ is determined by

$$M_v = e^{-\theta}\delta M_v + \sum_{k=1}^{\infty} e^{-\theta}\frac{\theta^k}{k!}\int_l^h kf(w)\left[1 - F(w)\right]^{k-1} M_v(w)dw. \tag{134}$$

The utility of a filled vacancy is

$$M_v(w) = 1 - w + \delta b M_v + \delta(1 - b)M_v(w). \tag{135}$$

Solving $M_v(w)$ and inserting it back to (134) yields the following formula

$$M_v = e^{-\theta}\delta M_v + \sum_{k=1}^{\infty} e^{-\theta}\frac{\theta^k}{k!} /_l^h - \left[1 - F(w)\right]^k (1 - \delta + \delta b)^{-1}\delta b M_v + \tag{136}$$

$$\sum_{k=1}^{\infty} e^{-\theta}\frac{\theta^k}{k!} /_l^h - \left[1 - F(w)\right]^k (1 - \delta + \delta b)^{-1} (1 - w) -$$

$$\sum_{k=1}^{\infty} e^{-\theta}\frac{\theta^k}{k!}\int_l^h \left[1 - F(w)\right]^k (1 - \delta + \delta b)^{-1} dw.$$

Finally, we can simplify this by doing the summations and by changing the order of the summation and integration in the last sum

$$M_v = (1 - \delta)^{-1}(1 - \delta(1 - b)e^{-\theta})^{-1}\left\{1 - l - (1 - h)\, e^{-\phi} - \int_l^h e^{-\theta F(w)}dw\right\} \tag{137}$$

The expected utility of a worker must be the same regardless of which element he chooses from the support of its mixed strategy. The utility of a worker that uses l is

$$M_u^l = l + \delta b M_u^l + \delta(1 - b)M_u(l) \tag{138}$$

and the utility of a worker that uses h is

$$M_u^h = e^{-\theta}\left[h + \delta b M_u^h + \delta(1 - b)M_u(h)\right] + (1 - e^{-\theta})\delta M_u^h \tag{139}$$

as offering the highest possible wage means that the wage is always accepted. Finally, if the vacancy offers wage w its utility is determined by

$$M_u^w = \sum_{k=0}^{\infty} e^{-\theta}\frac{\theta^k}{k!}\left[\left[1 - F(w)\right]^k (w + \delta b M_u^w + \delta(1 - b)M_u(w)) + \left(1 - (1 - F(w))^k\right)\delta M_u^w\right] \tag{140}$$

The utilities of being employed at a specific wage are

$$M_u(l) = l + \delta b M_u^l + \delta\left(1-b\right) M_u(l) \tag{141}$$

$$M_u(h) = h + \delta b M_u^h + \delta\left(1-b\right) M_u(h) \tag{142}$$

$$M_u(w) = w + \delta b M_u^w + \delta\left(1-b\right) M_u(w) \tag{143}$$

Solving from these the expected utility and inserting in formulae (138)-(140) and in turn forcing them to yield the same expected utility as auction, namely that given by (133) allows us to solve for the endpoints of the support of the mixed strategy as well as the mixed strategy itself

$$l = \frac{e^{-\theta}}{1-\delta(1-b)\theta e^{-\theta}}, \tag{144}$$

$$h = \frac{1-\delta(1-b)\left(1-e^{-\theta}\right)}{1-\delta(1-b)\theta e^{-\theta}}, \tag{145}$$

$$F(w) = \frac{1}{\theta}\ln\frac{w\left[1-\delta(1-b)\theta e^{-\theta}\right]-\delta(1-b)e^{-\theta}}{e^{-\theta}(1-\delta(1-b))}. \tag{146}$$

We denote the cumulative distribution function for the realised wages by $G(w)$ and the corresponding density function is denoted by $g(w)$.

$$G(w) = \frac{1-e^{-\theta F(w)}}{1-e^{-\theta}} \tag{147}$$

$$= \frac{1}{1-e^{-\theta}} - \frac{(1-\delta(1-b))e^{-\theta}}{\left(1-e^{-\theta}\right)\left[w(1-\delta(1-b)\theta e^{-\theta})-\delta(1-b)e^{-\theta}\right]} \tag{148}$$

The density function is

$$g(w) = \frac{\left(1-\delta(1-b)\theta e^{-\theta}\right)(1-\delta(1-b))e^{-\theta}}{\left(1-e^{-\theta}\right)\left[w(1-\delta(1-b)\theta e^{-\theta})-\delta(1-b)e^{-\theta}\right]^2} \tag{149}$$

12.1 Equilibrium of Dynamic Model 2

In equilibrium, workers are indifferent between sending applications and receiving offers from firms, and firms are indifferent between making offers and receiving applications from workers. That is, for a worker

$$\frac{e^{-\theta}}{(1-\delta)(1-\delta(1-b)\theta e^{-\theta})} = \frac{1-e^{-\phi}-\phi e^{-\phi}}{(1-\delta)\left(1-\delta(1-b)\phi e^{-\phi}\right)}, \tag{150}$$

and for a firm

$$\frac{e^{-\phi}}{(1-\delta)(1-\delta(1-b)\phi e^{-\phi})} = \frac{1-e^{-\theta}-\theta e^{-\theta}}{(1-\delta)\left(1-\delta(1-b)\theta e^{-\theta}\right)}. \tag{151}$$

The left-hand side are utilities from sending wage demands or offers, and the right-hand sides are utilities from receving them. It turns out that in equilibrium $\theta = \phi \equiv \theta_0 \approx 1.146$, and there exists a unique equilibrium for strictly positive x and y:

$$x = \frac{\theta_0 \left(\alpha\theta_0 - 1\right)}{\alpha\left(\theta_0^2 - 1\right)}, \tag{152}$$

$$y = \frac{\alpha\theta_0 - 1}{\theta_0^2 - 1}. \tag{153}$$

Plugging the above solutions for x and y into $m(w)$ and $g(w)$ and using the appropriate ranges for the distributions just like in the two models above, we get the equilibrium distribution for the realised wages. The density function has approximately the same shape as the one produced by the static model.

Steady state

In a steady state equilibrium the number of matches equals the number of separations:

$$yv(1-e^{-\theta}) + (1-x)u(1-e^{-\phi}) = b(E-V), \tag{154}$$

where $yv(1-e^{-\theta})$ is the number of matches that form in a market where firms are urns, and $(1-x)u(1-e^{-\phi})$ is the number of matches in the market where firms are urns, and $b(E-V)$ is number of matches that break down per period. In the next version of the paper we will study the effect of changes in the breakdown probability b on the composition of the market (that is, on x and y) and on the wage distribution.

13 References

Bontemps C., Robin J-M., and van den Berg G. (2000) "Equilibrium Search with Continuous Productivity Dispersion: Theory and Nonparametric Estimation" *International Economic Review* 41, 305-357.

Burdett., K. and D.T. Mortensen (1998) "Wage Differentials, Employer Size, and Unemployment" *International Economic Review* 39, 257-273.

DiNardo, J., Fortin, N.M. and T. Lemieux, T. (1996) “Labor Market Institutions and the Distribution of Wages, 1973-1992: A Semiparametric Approach” *Econometrica* 64, 1001-1044.

Kultti, K. (1999) “Equivalence of Auctions and Posted Prices” *Games and Economic Behavior* 27, 106-113.

Montgomery, J.K. (1991) “Equilibrium Wage Dispersion and Interindustry wage Differentials” *Quarterly Journal of Economics* 106, 163-179.

Mortensen, D.T (2000) “Equilibrium Unemployment with wage Posting: Burdett-Mortensen meets Pissarides” in H. Bunzel, B.J. Christiansen, P. Jensen, N.M. Kiefer and D.T. Mortensen (eds.), Panel Data and Structural Labor Market Models. Amsterdam, Elsevier, 2000.

Pissarides, C.A. (2000) “Equilibrium Unemployment Theory”, The MIT Press.

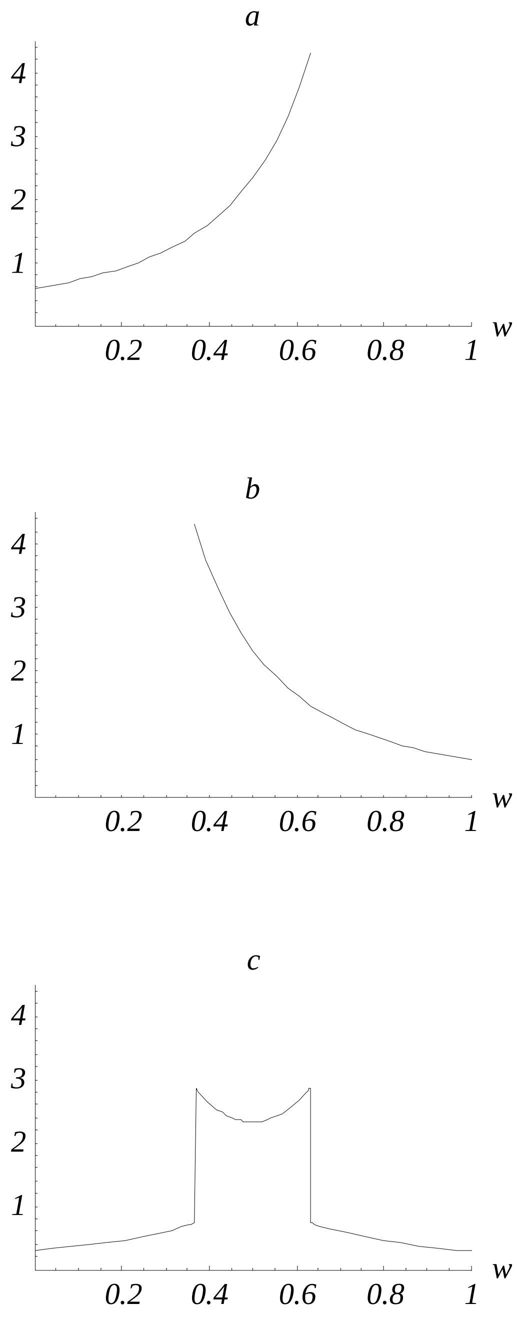

Figure 1: *Distributions of realized wages in the static model ($u/v = 1$). (a) Market where firms send offers to workers. (b) Market where workers send offers to firms. (c) Two-sided market.*

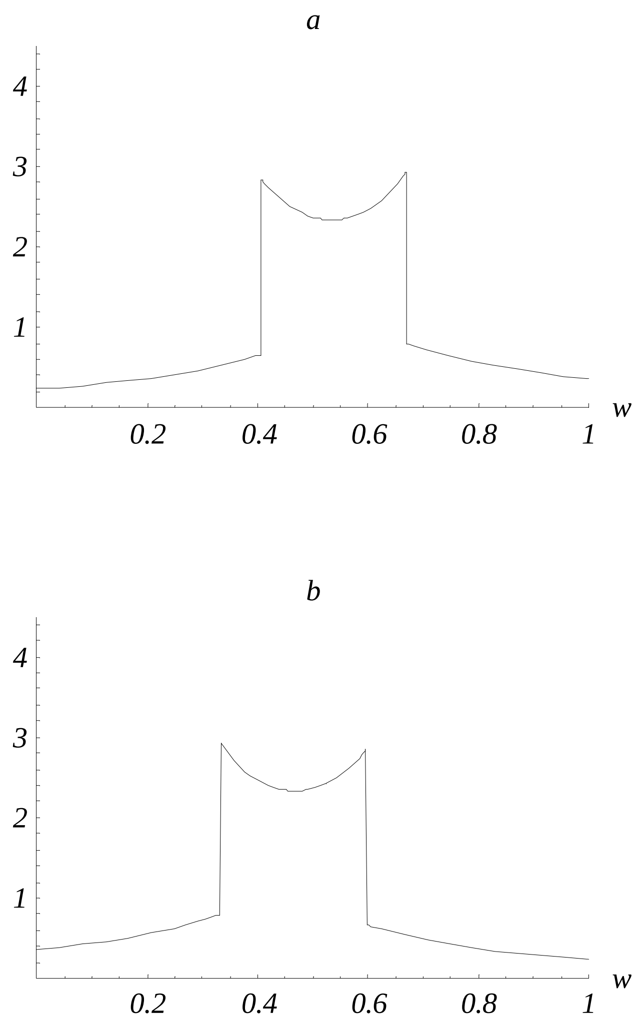

Figure 2: *Two-sided market in the static model, (a)* $u/v = 0.9$*, (b)* $u/v = 1.1$*.*

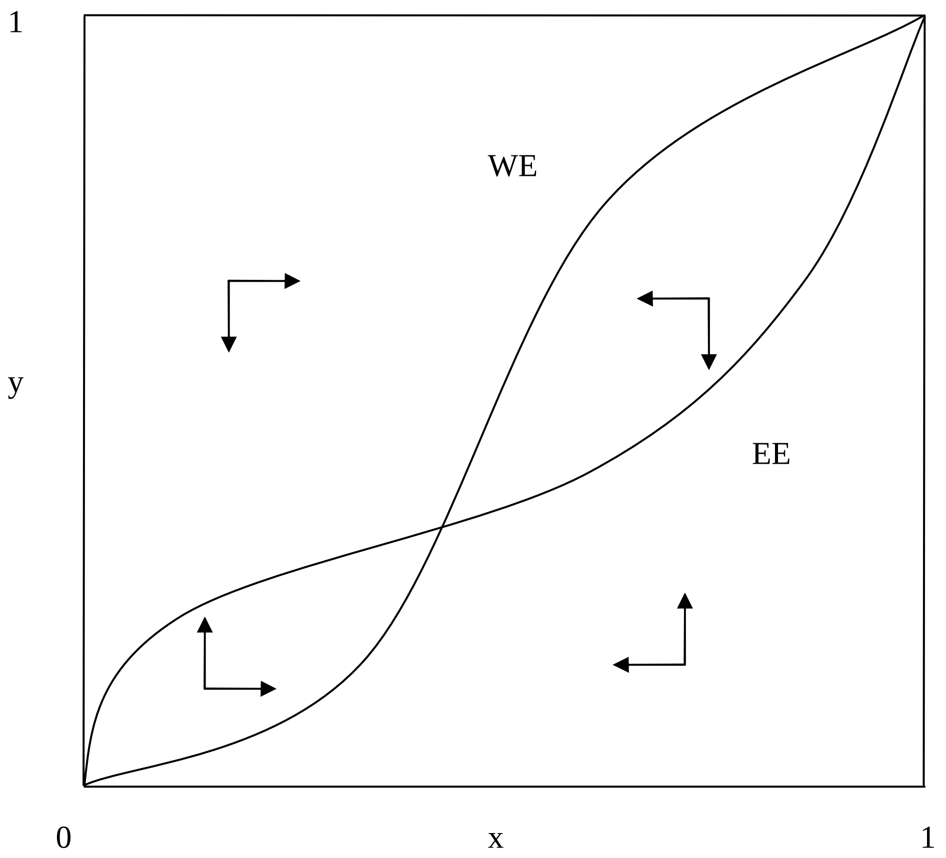

Figure 3: The mixed strategy equilibrium where $x \in (0,1)$ and $y \in (0,1)$ is stable.