An Experimental Investigation of the Explosive Effect in Almost Common Value Auctions

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Abstract

With only two bidders, a slight private value advantage in a common value auction is predicted to have an explosive effect on the outcome and revenue of the auction when using a second price and/or English clock mechanism. This paper uses the "wallet game" in an English clock mechanism with two players to test these predictions. The results show that the effect is proportional, not explosive, confirming the results of past experiments. A behavioral model with two types of players, naive and sophisticated, is developed and tested against the experimental data. In the model, naive players use a rule of thumb bidding function, while the sophisticated players are fully rational and, when forming their strategy (choosing the best response), correctly account for the probability that a rival is naive or sophisticated. We use cluster analysis to classify the players in the symmetric auctions as naive or sophisticated and a strict win/lose criteria to classify players in the asymmetric auctions. We find that those players classified as sophisticated are more aggressive in responding to high signals or advantaged status than their naive counterparts, but do not lower their bids relative to the naive players when receiving low signals or when regular.

1 Introduction

In a common value auction, the value of the object for sale is the same to all bidders. The typical example is an oil lease, where the amount of oil under the ground is the same regardless of who wins the lease. In an almost common value auction, one of the bidders is *advantaged* and values the object slightly more than the other *regular* bidders. In the oil lease example, one bidder may have developed a new, lower cost drilling method, giving that bidder a cost advantage. If this advantage is very small, it is not obvious that it will have an impact on the outcome or revenue generated by the auction as the advantaged bidder does not necessarily have the highest estimate (signal) of the object's value. But, in a startling result, Bikchandani (1988) showed that in a two-bidder second-price auction even a small epsilon advantage has an explosive effect on the outcome of an auction. The bidder with the advantage always wins and seller revenue is dramatically reduced.

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The possibility that one bidder (often with a known identity) may have an advantage over other bidders is not an unlikely scenario and these theoretical results may have important implications for auction design. Klemperer (1998) raises serious concerns about the use of ascending auctions for common value items since ascending auctions always reduce to two players in the end. His concerns are supported by anecdotal evidence such as the lower than expected revenue in the Los Angeles PCS license. Pacific Telephone is widely believed to have had an advantage due to its customer database and familiarity with the region. Entry costs further magnify Klemperer's concerns as regular bidders may avoid participating entirely. However, Avery & Kagel (1997), AK from here on, conducted an experiment on almost common value auctions with a sealed bid mechanism and found the effect of an advantage to be proportional, not explosive. The subjects in the AK study fell prey to the winner's curse even in the pure common value auctions. Specifically, they were employing a rule of thumb bidding function equivalent to bidding the unconditional expected value of the item without accounting for the adverse selection effect inherent in winning. Given that subjects failed to follow Nash equilibrium bidding in the symmetric game, perhaps it is not too surprising that the explosive effect failed to materialize in the "almost" common value game.

Our paper has two main objectives: the first is to present a behavioral model of the two-bidder almost common value auction. Bidders may be one of two types; naive or sophisticated, with each type representing a different level of intelligence. In our model, the effect of a small advantage is proportional, not explosive. The second objective is to move the analysis to an English clock auction.

The AK study documented deviation from the Nash equilibrium predictions by the bidders. Our first objective is motivated by a desire to explain the deviations. We test our model with an experiment designed to allow identification of naive and sophisticated players in the study. The second objective is motivated by the observation that behavior in ascending auctions is typically much closer to equilibrium outcomes than in corresponding sealed bid auctions.[1]

The use of auctions to sell publicly owned resources, such as spectrum rights, is growing. Spectrum auctions have raised billions of dollars worldwide, with some auctions, such as the British 3G auctions, far exceeding expectations and others, such as the Swiss UMTS Spectrum auctions, falling far short of expectations. (See Binmore & Klemperer, 2002 and Wolfstetter, 2001) Understanding the role that small asymmetries play in common value auctions has important mechanism design implications. Sorting out which bidders have an advantage, and determining the size of an advantage present in an auction is very difficult when using real-world data. This makes the problem ideal for a laboratory study where these variables can be controlled. We are unaware of any studies prior to this one that uses an English clock auction to study this issue.

The structure of this paper is as follows: Sections 2 and 3 explain the wallet game and summarize the AK findings. Section 4 presents our model. Section 5 explains the experimental design we used, while Section 6 describes our experimental hypotheses. We report the results in Section 7 and Section 8 concludes.

2 The Wallet Game

The wallet game is a pure common value auction where each of two players observes an independent random variable. Player 1 observes X and Player 2 observes Y. The two players then bid in a

[1] See the survey in Kagel and Levin's "Common Value Auctions and the Winner's Curse," Princeton University Press, 2002

second price auction for an object with value V, where $V = X + Y$. The unique, symmetric Nash equilibrium predicts that players bid twice their value.[2] The Nash equilibria of the wallet game, like all second-price auctions, rests on the willingness to pay argument. Players bid so that, in the event of a tie they are indifferent between winning and paying their own bid and losing the auction. In equilibrium, the winning bidder never makes negative profits.

In the almost common value version of the wallet game, one of the bidders has a private value advantage of K, where K is some positive number. The object is worth $V^* = X + Y + K$ to the advantaged bidder, and worth only $V = X + Y$ to the other, regular bidder. Here, the willingness to pay argument breaks down. There is no bid price at which both players are content to tie. The regular bidder knows that the advantaged bidder values the object slightly more, and since tie implies that the advantaged bidder is unwilling to pay any more the regular bidder must be losing money at that bid price. This creates an adverse selection problem for the regular bidder who thus wants to break the tie by lowering his bid. In contrast, the advantaged bidder, knowing that the regular bidder is not willing to pay the tieing bid, wants to break the tie by raising her bid. It is this breakdown that leads to the explosive effect demonstrated by Bikchandani. The Nash equilibrium predicts that the advantaged bidder will win the auction, and that the regular bidder will bid much lower than in the pure, symmetric common value version of the wallet game.[3]

3 Summary of the Avery & Kagel Paper

AK used the wallet game as the basis for their experiment with a sealed-bid second price auction. X and Y were distributed independently and uniformly over the range $[1, 4]$. They used two treatments. In the symmetric case, K was set to zero for both players. In the asymmetric case, K was set to 1 for one of the bidders.

Along with the Nash equilibrium predictions, AK also tested a behavioral model that may best be called the expected value model of bidding. For the symmetric case, this model predicts that players will bid their expected value given their signal. In the asymmetric auctions the expected value model predicts that advantaged players simply add the advantage to their bid. The bid function of the regular player in the asymmetric case remains unchanged relative to the symmetric case.

AK found that the expected value model provides a better fit of the data in both the symmetric and asymmetric cases than the Nash equilibrium predictions. The effect of the advantage on winning and revenue was proportional instead of explosive. The expected value model, however, does not fully explain the data. It is clear from the data that there is a great deal of heterogeneity among the players and AK found evidence to suggest that some of the players were best responding, as in Nash theory and in contrast to the expected value model.

In the next section we present a model that explicitly accounts for the heterogeneity of the players and allows us to further explore the possibility that players best respond.

[2] The wallet game has an infinite number of asymmetric equilibria (such as bully-sucker). See Milgrom (1981) for details regarding equilibria in common value second-price auctions.

[3] See Klemperer (1998) and Avery & Kagel (1997) for a more complete discussion of this equilibrium in the wallet game.

4 The Model

The AK study documented the players deviations from the theoretical predictions, and showed that the explosive effect does not occur if all players use a naive expected value bidding function. In addition, their results provide an example of heterogeneity among experimental subjects. Failure to achieve Nash equilibrium and heterogeneity among subjects are well documented phenomena in the experimental economics literature and have spurred efforts to develop models that account for heterogeneity through differing levels of sophistication.[4] Stahl (1993) models heterogeneity as a hierarchy of ever increasing levels of intelligence. At the bottom are unintelligent players that blindly follow a single fixed strategy, while at the very top are the superintelligent, standard Bayesian rational agents with an infinite number of types in between. Although there is likely a continuum of player types in the population, we focus on the two extremes, which we call naive and sophisticated. We then consider how allowing these two types changes the behavioral predictions.

The wallet game is used as the basis of our model. X and Y are drawn iid from a uniform distribution on $[0, 1]$. Let X be the signal received by the sophisticated player and Y be the signal received by the naive player. The value of the wallet is $V = X + Y$. Naive type players ignore the inference of information contained in the event of winning. Thus, they always use a 'naive' form of bidding, $B_N(Y) = \frac{1}{2} + Y$. This is the player's signal, Y, plus the unconditional expected value of the opponent's signal. We chose this bid function for the naive player for two reasons. First, it is a reasonable rule of thumb. The naive player simply fails to take the next logical step and consider what winning implies for the value of the opponent's signal. Second, this bid function is supported by past experiments and thus seems a reasonable bid function to expect players to use. The sophisticated type player is the standard, Bayesian rational agent. The sophisticated player adjusts her behavior to account for the possibility that her opponent is naive. How the behavior adjusts depends on the probability of facing a naive or sophisticated rival, and thus on the proportion of naive and sophisticated types in the population. This proportion is captured by the parameter $\lambda \in [0, 1]$. There are λ naive types and $(1 - \lambda)$ sophisticated types in the population. We model behavior from the point of view of the sophisticated player, as a naive player is 'hard wired' to bid $B_N(Y)$, and consider how behavior changes with respect to λ.

4.1 The Symmetric Auction

First, we calculate the best response bid function of the sophisticated player when she knows her opponent is naive ($\lambda = 1$.) The naive player's bid function is $B_N(Y) = \frac{1}{2} + Y$. The sophisticated player conditions her bid on winning, and considers her opponent's bid function. With a bid of B, the sophisticated player wins whenever $B_N(Y) = \frac{1}{2} + Y \leq B$, or $Y \leq B - \frac{1}{2}$. Thus, the sophisticated player who observes $X = x$ solves:

$$\underset{B}{Max} \int_0^{B-\frac{1}{2}} [(x + t) - (\frac{1}{2} + t)]dt \tag{1}$$

Differentiating with respect to B yields

$$[x + (B - \frac{1}{2})] - [\frac{1}{2} + (B - \frac{1}{2})] = x - \frac{1}{2} \lesseqgtr 0, \ as \ x \lesseqgtr \frac{1}{2} \tag{2}$$

[4] For a survey of examples, see Camerer, 2003 Chapter 5.

We know that when $Y = 1$, $B_N(Y) = \frac{3}{2}$. If $X \geq \frac{1}{2}$ then the expected value of the wallet to the sophisticated player is greater than the naïve type player's bid: $V = X + Y \geq Y + \frac{1}{2}$. The sophisticated player earns a positive profit of $X - \frac{1}{2}$. Therefore, when the sophisticated player receives a signal $X \in [\frac{1}{2}, 1]$ she bids to ensure she wins using the following bid function:

$$B_s(X) = \left\{ \begin{array}{lll} \frac{3}{2} & if & \frac{1}{2} \leq X \end{array} \right. \tag{3}$$

Conversely, when the sophisticated player receives a signal $X \in [0, \frac{1}{2})$, her expected value of the wallet is less than the naive player's bid. The naive player's behavior creates a winner's curse problem for the sophisticated player in this range. The sophisticated player does not want to win for such signals, and bids low enough to be sure she loses. Therefore,

$$B_s(X) = \left\{ \begin{array}{lll} X & if & X < \frac{1}{2} \end{array} \right. \tag{4}$$

The sophisticated player switches between wanting to lose and wanting to win when her expected profit switches from negative to positive.

When the sophisticated player knows that her opponent is also sophisticated ($\lambda = 0$), she uses the symmetric Nash bid function $B_S(X) = 2X$. The naive player's bid function and the sophisticated player's best responses to players of known type are shown in Figure 1.

We now present the equilibrium of the game when $\lambda \in (0, 1)$ and we let $\delta = \frac{1-\lambda}{\lambda}$.

[INSERT FIGURE 1 ABOUT HERE]

Proposition 1 *The unique symmetric equilibrium is given by*[5]:

$$B_S(X) = \left\{ \begin{array}{lll} 2X & if & x \leq \frac{1}{4} \\ \frac{\delta}{\delta-1}2X + \frac{1}{\delta-1}[\frac{1}{2}]^{1-\delta}(1-2X)^{\delta} - \frac{1}{\delta-1} & if & \frac{1}{4} < X \leq \frac{1}{2} \\ \frac{\delta}{\delta-1}2X - \frac{1}{\delta-1}[\frac{1}{2}]^{1-\delta}(2X-1)^{\delta} - \frac{1}{\delta-1} & if & \frac{1}{2} \leq X < \frac{3}{4} \\ 2X & if & \frac{3}{4} \leq X \leq 1 \end{array} \right. \tag{5}$$

and illustrated in Figure 2.
Proof. *See Appendix A, Part I* ■

First note that: $B_S(\frac{1}{2}) = 1$; the $\lim_{\frac{1}{4}<X\to\frac{1}{4}} B'_S(X) = \lim_{\frac{3}{4}>X\to\frac{3}{4}} B'_S(X) = 0$,and the $\lim_{\frac{1}{2}>X\to\frac{1}{2}} B'_S(X) = \lim_{\frac{1}{2}<X\to\frac{1}{2}} B'_S(X) = \infty$, so that the solution does indeed look like Figure 2. Next, note that when $\lambda = \frac{1}{2} \iff \delta = 1$ and we seem to be dividing by $(\delta - 1) = 0$. However, $\delta = 1$ implies that the numerator in equation 5 vanishes as well. Applying L'Hopital's rule we derive for $\lambda = \frac{1}{2}$ ($\delta = 1$) that

$$B_S(X) = \left\{ \begin{array}{lll} 2X + (1-2X)\ln(2-4X) & if & \frac{1}{4} \leq X \leq \frac{1}{2} \\ 2X + (1-2X)\ln(4X-2) & if & \frac{1}{2} \leq X \leq \frac{3}{4} \end{array} \right. \tag{6}$$

Next, we sketch the economic reasoning behind the proof. If the sophisticated player's signal falls within Region I or Region III of Figure 2, ($X \in [0, \frac{1}{4}]$ or $X \in [\frac{3}{4}, 1]$), then the sophisticated

[5] As mentioned before, we do not discuss the equilibrium bid function of the naive player since the naive player is hard-wired to bid $B_N(Y) = \frac{1}{2} + Y$

player knows that a tieing bid can only come from another sophisticated player. This is because the naive player never bids less than $\frac{1}{2}$ in Region I, and never bids more than $\frac{3}{2}$ in Region III. Using the maximum willingness to pay argument, the sophisticated player uses the symmetric Nash equilibrium bid function $B(X) = 2X$. When the sophisticated player's signal falls within Region II, a tieing bid could come from either a naive player or another sophisticated player. If the signal falls in the range $(\frac{1}{4}, \frac{1}{2})$ the possibility of a tie with a naive player causes the sophisticated player to bid more cautiously than if she knew her opponent was sophisticated. This is illustrated as the increasing curve in Region II of Figure 2. The sophisticated player bids cautiously because if the tieing bid comes from a naive player, the value of the object will be less than $2X$, where X is the sophisticated player's signal. The possibility of a tie coming from another sophisticated player, however, causes the sophisticated player to bid more aggressively than if she knew her opponent was naive. If the signal is in the range $(\frac{1}{2}, \frac{3}{4})$, the possibility of tieing with a naive player makes the sophisticated player bid more aggressively than against another sophisticated player, but less aggressively than against a known naive type player. This is illustrated as the decreasing curve in Region II of Figure 2. The sophisticated player bids more aggressively than against a known sophisticated opponent because if the tie is from a naive player the value of the object is greater than $2X$. The possibility that the tie does come another sophisticated player disciplines the bidding, and makes it less aggressive than against a known naive player. How cautious a sophisticated player is for signals in the range $(\frac{1}{4}, \frac{1}{2})$, and how aggressive a sophisticated player is for signals in the range $(\frac{1}{2}, \frac{3}{4})$ will depend on the value of λ. When λ is small, the curves move toward the Nash equilibrium, when λ is large, the curves become flatter. This analysis implies that sophisticated players will lose more frequently than naive players for signals below $\frac{1}{2}$, and win more frequently than naive players for signals above $\frac{1}{2}$.

[INSERT FIGURE 2 ABOUT HERE]

4.2 The Asymmetric Auction

Now suppose that one player has a private value advantage of K, where $K \in [0, \frac{1}{2})$. The value of the wallet to the advantaged player is $V^A = X + Y + K$, while the value to the regular player remains $V^R = X+Y$. The situation is much more complicated than the symmetric case because the sophisticated player may be advantaged or regular and her opponent may be naive or sophisticated. This yields four possible cases that must be considered: a sophisticated regular player versus a naive advantaged player; a sophisticated advantaged player versus a naive regular player; a sophisticated regular player versus a sophisticated advantaged player and a sophisticated advantaged player versus a sophisticated regular player.[6]

As in the symmetric case, we first discuss a sophisticated player's best response to an opponent of known type ($\lambda = 1$ or $\lambda = 0$), and then discuss what happens when $\lambda \in (0, 1)$.

If the sophisticated player's opponent is also sophisticated ($\lambda = 0$), the result is the explosive Nash equilibrium. The advantaged player wins all the auctions and increases her bid by more than the amount of the advantage. The regular sophisticated player reduces his bid relative to the symmetric auction. Below, we first discuss the case in which a sophisticated player is regular, and then turn our attention to the case in which the sophisticated player is advantaged.

[6] We do not discuss the possibility of a naive advantaged (regular) player versus a naive regular (advantaged) players as the naive player's bid function is hard-wired.

We assume that the naive player is "hard wired" to use the expected value model. When the naive player is regular, he bids $B_N^R(Y) = \frac{1}{2} + Y$. When advantaged, the naive player adds the value of K to his bid, resulting in a bid of $B_N^A(Y) = \frac{1}{2} + Y + K$.

4.2.1 Case 1 - Sophisticated Regular versus Naive Advantaged

The naive advantaged player adds the private value component to his bid: $B_N^A(Y) = \frac{1}{2} + Y + K$. For the sophisticated regular player to win, $B_S^R(X) \geq B_N^A(Y)$ must be true. Thus $B_S^R(X) \geq \frac{1}{2} + Y + K$ and $Y \leq B_S^R(X) - \frac{1}{2} - K$. The sophisticated regular player's problem is:

$$\underset{B}{Max} \int_0^{B-\frac{1}{2}-K} [(x+t) - (\frac{1}{2} + t + K)]dt \tag{7}$$

Differentiating with respect to B yields:

$$[x + (B - \frac{1}{2} - K)] - [\frac{1}{2} + (B - \frac{1}{2} - K) + K] = x - \frac{1}{2} - K \lesseqgtr 0, \ as \ x \lesseqgtr \frac{1}{2} + K \tag{8}$$

When the naive player is advantaged, we know that if $Y = 1$, then $B_N^A = \frac{3}{2} + K$. When $X > \frac{1}{2} + K$ the value to the sophisticated regular bidder is $V = X + Y > \frac{1}{2} + K + Y$. The sophisticated regular player earns a positive profit of $X - (\frac{1}{2} + K)$. Therefore, from our previous discussion of the symmetric case, the bid function of the sophisticated regular player is

$$B_S^R(X) = \begin{cases} X & if \quad X \in [0, \frac{1}{2} + K) \\ \frac{3}{2} + K & if \quad X \in [\frac{1}{2} + K, 1] \end{cases} \tag{9}$$

In contrast to the case with two sophisticated players, the regular player no longer wants to lose the auction for all signal values. In the symmetric case ($K = 0$), the sophisticated regular player wants to lose whenever $X \in [0, \frac{1}{2})$, because expected profit is negative. (See Figure 1.) For signal values above $X = \frac{1}{2}$, the expected profit of the sophisticated player becomes positive. A positive value of K raises the point at which the sophisticated player's expected profit switches from negative to positive. We illustrate this in Figure 3 using $K = \frac{1}{3}$. The effect of the advantage on the outcome of the auction is proportional to the size of K, not explosive. As K increases to $\frac{1}{2}$ the point at which the sophisticated player's expected profit switches from negative to positive moves to the right and the region in which the sophisticated regular player wants to win shrinks to nothing. When $K \geq \frac{1}{2}$, the explosive effect is restored; the advantaged bidder wins all the auctions and revenue decreases relative to the symmetric case with two sophisticated players.

[INSERT FIGURE 3 ABOUT HERE]

4.2.2 Case 2 - Sophisticated Advantaged versus Naive Regular

The naive regular player's bid function is $B_N^R(Y) = \frac{1}{2} + Y$ (the hard-wired bid function). The sophisticated advantaged player wins when $B_S^A(X) \geq B_N^R(Y) = \frac{1}{2} + Y$ or $B - \frac{1}{2} \geq Y$. The sophisticated advantaged player's problem becomes:

$$\underset{B}{Max} \int_0^{B-\frac{1}{2}} [(x+t+K)-(\frac{1}{2}+t)]dt \tag{10}$$

Differentiating with respect to B yields

$$[x+(B-\frac{1}{2})+K]-[\frac{1}{2}+(B-\frac{1}{2})] = x+K-\frac{1}{2} \lesseqgtr 0, as\ x \lesseqgtr \frac{1}{2}-K \tag{11}$$

It is clear from the previous discussions that the sophisticated player will bid

$$B_S^A(X) = \begin{cases} X+K & if \quad X \in [0, \frac{1}{2}-K] \\ \frac{3}{2} & if \quad X \in (\frac{1}{2}-K, 1] \end{cases} \tag{12}$$

This is the reverse of the sophisticated regular versus naive advantaged case. The value of K lowers the point at which the expected profit of the sophisticated advantaged player switches from negative to positive relative to the symmetric case. Even though the sophisticated advantaged player values the object more than the naive regular player, there is a region of signals where the sophisticated regular player does not want to win the object. We illustrate this in Figure 4 using $K = \frac{1}{3}$. When $K < \frac{1}{2}$, the effect of the advantage on the outcome of the auction is proportional. As K increases the point at which expected profit switches from negative to positive moves to the left and this region becomes smaller. When $K \geq \frac{1}{2}$, the region in which the sophisticated advantaged player wants to lose disappears, and the explosive effect is restored.

[INSERT FIGURE 4 ABOUT HERE]

4.2.3 Cases 3 & 4 - Sophisticated versus Sophisticated

When a sophisticated advantaged player faces a sophisticated regular player, we are back in the case described by AK in their paper and we paraphrase their argument. Here, the advantaged player wins all the auctions. If this were not so, there must be a case in which $B_S^A(X) = B_S^R(Y)$. But, the advantaged player knows the object is worth K more to her than to her opponent. Thus, if the regular player is willing to pay $B_S^A(X) = B_S^R(Y)$, the advantaged player wants to increase her bid to insure winning. The regular player also knows that the object is worth K more to the advantaged player and so wants to reduce his bid to break the tie. Thus the sophisticated advantaged player bids more than the maximum possible value to her opponent and the sophisticated regular player bids his minimum possible value. With signals drawn from the $[0,1]$ interval, the sophisticated advantaged player's bid function is

$$\frac{3}{2} + X \geq B_S^A(X) \geq X+1 \tag{13}$$

The sophisticated advantaged player's bid function is bounded from above by $\frac{3}{2}+X$ since any higher bid is weakly dominated. The sophisticated regular player bids his minimum possible value

$$B_S^R(Y) = Y \tag{14}$$

as any bid smaller than this is dominated.

4.2.4 Sophisticated Player is Uncertain of Opponent's Type

Now we turn our attention to what happens when $\lambda \in (0,1)$. As noted above, this is far more complicated than the symmetric case. Here, all four possible types of players (sophisticated advantaged, sophisticated regular, naive advantaged, naive regular) have distinct bid strategies that are functions of both their type and signal. Thus, the four cases must be solved simultaneously yielding a system of differential equations. The strategies of the naive types (regular or advantaged) are "hard-wired". Therefore, we discuss the solution solely from the point of view of the two sophisticated types.

Denote by $B_N^R(X)$, $B_N^A(X)$, $B_S^R(X)$, and $B_S^A(X)$ the bidding strategies of a naive regular, naive advantaged, sophisticated regular, and sophisticated advantaged bidder respectively.

Proposition 2 *The equilibrium bidding with the $K = \frac{1}{2}$ used in our design, is given by:*
$B_N^R(X) = \frac{1}{2} + X$; $\quad B_N^A(X) = 1 + X$,
$B_S^R(X) = X$, *and* $\frac{3}{2} \leq B_S^A(X) \leq (\frac{3}{2} + X)$. *See the appendix for discussion of the more general case of* $K \in [0, \frac{1}{2})$
Proof. *See Appendix A, Part II* ■

As K decreases, the region in which the advantaged (regular) player wants to lose (win) becomes larger. As K increases, the region becomes smaller. Both the size of this region and the effect on the outcome of an auction are proportional to the size of the advantage. The effect of an advantage on both winning and revenue is proportional for $K \in [0, \frac{1}{2})$, and only becomes explosive when $K \geq \frac{1}{2}$.

This is a second price auction, and as discussed earlier, the Nash equilibrium solution relies on a willingness to pay argument. Consider a case in which the two bidders tie. The regular player knows that the other player is advantaged and that the object is thus worth more to his opponent. If the tieing bid is the most the advantaged player is willing to pay for the object, then the regular player cannot be willing to pay that amount and wants to lower his bid.

Conversely, the advantaged bidder realizes that if the regular player is willing to pay the tieing bid price, then the object must be worth that amount to the regular player. This means the object is worth even more to the advantaged player and she would like to increase her bid to break the tie. Thus, in the almost common value case, there can be no tie between the regular and advantaged players. This is what causes the explosive effect. In our model, the sophisticated player's uncertainty about her opponent's type mitigates the explosive effect for 'small' K. The design used in our experiment restores this explosive effect by using a 'large' value of K.

5 Experimental Design

This experiment uses the "wallet game" model. The private signals of the bidders, X and Y, are distributed independently and uniformly over $[1, 4]$. These parameters were deliberately chosen to allow comparison to the sealed bid experiment done by AK. For the asymmetric case we set $K = \$1.50$. This is equivalent to setting $K = \frac{1}{2}$ when the signals are distributed over $[0, 1]$. This level of K restores the explosive effect by insuring that there is no region of signals in which a sophisticated advantaged player would want to lose, or a sophisticated regular player would want to win, such as those shown on Figures 3 & 4. Using such a large advantage creates a stark contrast between advantaged and regular players and should help players understand that winning without the advantage leads to losses. This should aid in classifying the players as naive or sophisticated.

We ran a total of four sessions. Two sessions of the standard symmetric auctions and two of the asymmetric auctions[7]. See Table 1. Subjects were recruited by e-mail from a list of students registered for economics undergraduate courses at The Ohio State University in the past year. They were told the sessions would last approximately two hours, and guaranteed a minimum payment of at least $6.00.

[INSERT TABLE 1 ABOUT HERE]

The subjects were given a $12.00 starting balance which included their $6.00 participation fee. Profits and losses incurred during the session were added or subtracted from their balances and paid to them in cash at the end of the session. Average payments in the symmetric sessions were $16.37 and $27.33 in the asymmetric sessions. All subjects received more than the minimum payment of $6.00. In each session we conducted three practice auctions to allow subjects to become familiar with the software and to give them an opportunity to ask questions regarding the procedures. This was followed by twenty-five auction rounds played for cash. To account for possible trial and error at the beginning of the sessions, we dropped the first five rounds of each session from our analysis.

At the beginning of each round, the computer software randomly and anonymously paired the subjects and generated their private signals. In the asymmetric auctions, the computer also randomly determined which player in each pair was advantaged. The random assignment of the advantage means that players were not necessarily advantaged or regular an equal number of times during the experiment. The players then participated in an English clock auction with the common value determined by the sum of their private signals. In all cases, the new signal along with the possible value range of the object based on the signal appeared on the players' screens 15 seconds prior to the start of each auction. The clock flashed 10 times and then started at $2.00 and counted up by pennies. The players were instructed to drop out of the auction when the clock reached the highest price they were willing to pay. As soon as one player in each pair dropped out of the auction, the auction ended. The players in the asymmetric auctions knew the value of K and knew if they were advantaged or regular in each auction round.

At the end of each auction, the results were displayed on the players' computer screens. They were given the following information for auctions in which they participated: the common value of the object, the signals of both players, the drop out price, and the high bidder's profit. The high bidder's bid was displayed as XXX since the actual value the player was willing to bid was unknown. The value of K for a particular bid, either zero or $1.50, was included in the results given to the players in the asymmetric auctions and displayed next to the signals. Players were given 30 seconds to review the results before new information for the next round was displayed. Players could, if they wished, review all of their past auctions by scrolling through a history section displayed on their computer screens.

6 Experimental Hypotheses

The purpose of our experiment is to determine if it is possible to identify the percentage of players that are naive or sophisticated, and to determine if they use differing bid functions. We first test

[7] When there was an odd number of players, 1 player in each round was randomly selected by the computer to sit out.

the Nash equilibrium and expected value models as they provide natural benchmarks for evaluating the data. Then we identify the naive and sophisticated players and test how well our model explains the data.

In the symmetric auctions, the Nash equilibrium predicts that bidders bid twice their signals, $B(X) = 2X$. The expected value model predicts that players bid their signals plus the unconditional expected value of their opponent's signal, or $B(X) = 2.5 + X$ for the parameters used in our design.

Our naive/sophisticated model predicts that naive type players use the expected value bidding function, while the sophisticated type players best respond. In the symmetric auctions, this implies that the slope of the bid function for naive players is one, while the slope of the sophisticated players' bid function is two in Regions I & III shown on Figure 2. The slope of the sophisticated player's bid function in Region II of Figure 2 depends on the signal and the proportion of types in the population. Our model also makes predictions regarding the outcomes of the symmetric auctions. Sophisticated players should lose more frequently than naive players when receiving a signal below 2.5 ($\frac{1}{2}$ on the $[0, 1]$ interval) and should win more frequently than naive players when receiving a signal above 2.5.

In the asymmetric auctions, the Nash equilibrium model predicts that the advantaged bidder will win all the auctions, and the regular bidder will lower his bid relative to the symmetric auction. The expected value model predicts that the regular players will bid their signals plus the unconditional expected value of their opponent's signal. The advantaged players simply add the value of the advantage to their regular bid, or $B(X) = 2.5 + X + 1.5$ for the parameters used in our design. Which player (advantaged or regular) wins the auction in the expected value model depends on the realization of the private signals.

Our model predicts that sophisticated players will win all auctions in which they are advantaged and lose all auctions in which they are regular. Naive players will use the expected value bid function, and may win or lose auctions depending on the signals received and whether or not their opponent is sophisticated or naive. The revenue of the auction will also depend on the type of players in each auction. Sophisticated players will lower their bids relative to the symmetric auction, but naive players will not.

7 Experimental Results

7.0.5 Aggregate Results

Symmetric Auctions Before testing our model of naive and sophisticated type players, we first test the Nash equilibrium and expected value models. We begin with the symmetric auctions to establish a benchmark for comparison with the asymmetric auctions. The Nash equilibrium predicts a bid function of $B(X) = 2X$, while the expected value hypothesis predicts $B(X) = 2.5 + X$.

We are using an English clock mechanism with just two players and the auction ends as soon as one player drops out. Thus, we do not know the high bidder's exact bid, only that it is higher than the drop out price. This drop out price varies by auction. We accommodated the varying drop out prices by specifying a variable threshold Tobit model with random effects throughout our analysis. We specified:

$$B_{it} = \alpha_0 + \alpha_1 X_{it} + \epsilon_{it} \quad i = \{1, 2, ...N\} \;\; t = \{1, 2, ...T\} \tag{15}$$

The results for all the auctions and for the last 10 rounds as well as the tests of the expected value model ($\alpha_0 = 2.5$, $\alpha_1 = 1$) and the Nash equilibrium model ($\alpha_0 = 0$, $\alpha_1 = 2$) are shown in Table 2. We can clearly reject both the Nash equilibrium and the expected value models both for all the rounds and for the last 10 rounds. However, the results are much closer to the expected value predictions than to the Nash equilibrium.

This is further supported by examining the profits of the bidders. Winning bidders lost money in 35.31% of the symmetric auctions. This is inconsistent with the Nash equilibrium in which bidders never lose money. It is consistent with the expected value model as those bidders with signals less than 2.5 are expected to overbid and fall prey to the winner's curse, while those with signals above 2.5 are expected to underbid and earn positive profits. Noting that those bidders that won with a signal less than 2.5 lost money 84.52% of the time, while those with signals above 2.5 lost money only 17.8% of the time further reinforces our conclusion that the results are more consistent with the expected value model than with the Nash equilibrium model.

[INSERT TABLE 2 ABOUT HERE]

That bidders lost money clearly indicates that the subjects were overbidding and suffering from the winner's curse. We chose to use an English clock auction because, as noted above, it has helped mitigate the worst effects of the winner's curse in past experiments and we hoped to get a better test of the explosive effect. We combined our data with the AK inexperienced data to see if switching to the English clock mechanism reduced the winner's curse for the subjects. We specified three variable threshold Tobit models with random effects. The first is as follows:

$$B_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 M_{it} + \epsilon_{it} \;\; i = \{1, 2, ...N\} \; t = \{1, 2, ...T\} \tag{16}$$

where $M_{it} = 1$ for the English clock mechanism and $M_{it} = 0$ for the sealed bid second price mechanism used by AK. If the English clock mechanism is effective in reducing the winner's curse, the coefficient for the mechanism dummy should be negative. The results are shown in Table 3. The mechanism dummy variable has the wrong sign, is very small and is statistically insignificant.

We next specified:

$$B_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 MX_{it} + \epsilon_{it} \;\; i = \{1, 2, ...N\} \; t = \{1, 2, ...T\} \tag{17}$$

where MX_{it} is an interaction term created by multiplying M_{it} by the signal X_{it}. The expected sign of the coefficient is positive, as the English clock mechanism should make the players more responsive to their signal. The interaction term is positive, but small and statistically insignificant as shown in Table 3. Our final specification included both the mechanism dummy and the interaction term as follows:

$$B_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 M_{it} + \beta_3 MX_{it} + \epsilon_{it} \;\; i = \{1, 2, ...N\} \; t = \{1, 2, ...T\} \tag{18}$$

Neither term is statistically significant, nor are they jointly significant. The results are also shown in Table 3.

[INSERT TABLE 3 ABOUT HERE]

There are two possible explanations for the failure of the English clock mechanism to mitigate the winner's curse in our experiment. First, the structure of the game used requires players to bid above their signal value. This may be confusing and counterintuitive to the subjects, causing them to overbid. Second, English clock auctions with three or more bidders reveal more information than their sealed bid counterparts. Knowing the drop out prices of other bidders can allow the remaining bidders to update their beliefs about the value of the object and adjust their bids accordingly. Our design used only two bidders in each auction. Thus, as soon as one bidder drops out, the auction ends. In this case, the English clock mechanism does not reveal any additional information relative to the sealed bid auction.

Asymmetric Auctions In the asymmetric auctions, the explosive Nash equilibrium predicts that the advantaged players will win 100% of the auctions. Given the actual signals used in the experiment, the expected value model predicts that advantaged bidders will win 85.7% of the auctions. Instead, the advantaged players won only 75.6% of the auctions.

Under the expected value hypothesis, the slope of the bid function does not vary between advantaged and regular players; only the intercept changes. To test this we specified

$$B_{it} = \gamma_0 + \gamma_1 X_{it} + \gamma_2 D_{it} + \epsilon_{it} \tag{19}$$

Where $D_{it} = 1$ if the player is advantaged and $D_{it} = 0$ otherwise. The expected value model predicts that $\gamma_2 = 1.5$, and while the Nash equilibrium predicts that $\gamma_2 = 3$. The results for are shown in Table 4. We reject both, but as in the symmetric auctions, the results are clearly closer to the expected value hypothesis, and provide further evidence against the Nash equilibrium. Interestingly, the coefficient for the dummy variable is less than the value of K.

[INSERT TABLE 4 ABOUT HERE]

We next looked for the explosive effect relative to the symmetric auctions. The explosive effect predicts that the regular bidders will dramatically reduce their bids relative to the symmetric auctions. The Nash equilibrium also predicts that revenue will decrease dramatically in the asymmetric auctions. The average drop out price in the symmetric auctions was \$4.75, which is the same average drop out price found in the asymmetric auctions. These averages are drop out prices that do not account for any differences in the actual signals received. Before rejecting the explosive Nash hypothesis, we pooled the data from the symmetric and asymmetric auctions and compared the behavior in the symmetric auctions with those of the regular bidders in the asymmetric auctions. We specified

$$B_{it} = \gamma_0 + \gamma_1 X_{it} + \gamma_2 T_{it} + \epsilon_{it} \tag{20}$$

where $T_{it} = 1$ indicates the asymmetric auctions. The Nash equilibrium predicts that $\gamma_2 < 0$, while the expected value model predicts that $\gamma_2 = 0$. The results are reported in Table 5. The treatment dummy is significant and has the correct sign, but it is small. Regular players' bids decrease by only \$0.32 for a mean signal of 2.5. While the expected value model does a better job explaining the data than the Nash equilibrium model, it does not fully explain the data. We have established that the explosive effect does not exist in the aggregate data of our asymmetric auctions. We now turn our attention to testing our naive/sophisticated model. In order to test our model, we must first classify players as naive or sophisticated.

[INSERT TABLE 5 ABOUT HERE]

7.0.6 Classifying the Players

Overview of Cluster Analysis Economists generally focus on groups when describing economic behavior. We focus on groups because we expect the behavior of a member of group to be more like the behavior of other members of the same group than like the behavior of members of a different group. It is far more interesting and insightful to discuss differences between groups than to discuss differences between individuals.

In most situations it is easy to define groups and classify observations on the basis of cutoff points or intervals (e.g. age or income). But, suppose we would like only a few distinct groups. If we classify individuals based on socioeconomic variables, some individuals may have membership in more than one group. It would be better to group the individuals based on their overall similarity, but deciding which individuals (or observations) are most similar may be difficult to determine, especially if a large number of characteristics are used. This is where cluster analysis is useful.

Cluster analysis is a set of techniques that are designed to identify groups and classify observations into those groups based on similarity of characteristics[8]. If we are classifying individuals on the basis of three characteristics, we could represent each individual as a point in a three dimensional space, with each dimension reflecting a different characteristic. (For example, age, income, and education). After we have plotted all the individuals in this space, clusters of individuals may emerge, and we could simply eyeball the data and say this is group A and that is group B. Cluster analysis formalizes this process and forces us to define the criteria used to assign group membership. The goal is to minimize the differences within a group and maximize the differences between groups.

A subset of cluster analysis (and the type used in this study) is partitioning. The researcher chooses the number of clusters to be formed, and also chooses a distance measure (e.g. euclidean). The data is divided into the chosen number of groups, randomly or through some predetermined method and the means of the relevant characteristics for each group are calculated. Next, each observation is examined and its distance from each group is calculated using the distance measure. The observation is then reassigned to the group it is nearest. Means are recalculated, and the process repeats until no observations are reassigned.

We used Stata's "cluster kmeans" command with the euclidean distance measure to classify our subjects. The Stata algorithm splits the observations randomly into the specified number of groups; in our case, two. It calculates the group means and then compares each observation to the means and reassigns it to the group with the closest means. After all the observations have been examined and reassigned, Stata recalculates the group means. Recalculating the group means after all the observations have been examined ensures that the order in which the observations are examined does not matter. The process repeats until it completes a pass without reassigning any observations. (Stata Reference Manual, pg. 265)

Classifying the Symmetric Auction Subjects Our model defines players as naive or sophisticated based on their bidding behavior. Naive players use the expected value bid function, while sophisticated players use the bid function specified in Proposition 1. The sophisticated player's bid function is complicated and depends on the region from which the signal is drawn. But, if the signal is drawn from Region I or Region III ($x < \frac{1}{4}$ or $x > \frac{3}{4}$) then the sophisticated bid function is the same as the Nash bid function (as shown on Figure 2). Moreover, the Nash bid function can serve as a linear approximation of the bid function for signals drawn from Region II of Figure 2.

[8] See Lorr, 1983 for a survey of cluster analsysis techniques.

We have between 17 and 20 observations for each individual subject. Rather than estimating two regressions (one for signals drawn from Regions I & III and one for signals drawn from Region II of Figure 2) we used the Nash bid function as a proxy for the sophisticated bid function.

We used a variable threshold Tobit regression to estimate each individual subject's bid function. The goal is not to do hypothesis testing using these coefficients; they are used solely to classify the subjects as naive or sophisticated. We specified:

$$B_t = \alpha_0 + \alpha_1 X_t + \epsilon_t \tag{21}$$

We defined two criteria for clustering the players: the naive criteria and the sophisticated criteria. The naive criteria is calculated by subtracting the estimated coefficient from the expected value bid function's predicted coefficient for both the constant and the slope. The sophisticated criteria is calculated in the same manner, but using the predicted coefficients of the Nash bid function. This is summarized in Table 6. Once we had calculated the clustering criteria variables, we ran the cluster analysis twice. First using the naive criteria and then using the sophisticated criteria. Each time we ran the cluster analysis, two groups were created. The membership of the two groups created by both the naive and sophisticated criteria matched. There are 10 subjects in one group and 23 in the other. We then used the means of all four variables (shown in Table 7) to determine which group was naive and which was sophisticated. We have classified the group with 10 subjects, or 30%, as sophisticated.

[INSERT TABLES 6 & 7 ABOUT HERE]

It is important to note that cluster analysis can not tell us how many groups there should be, this must be decided based on theory or some other criteria. Cluster analysis only tells us that if there are two groups, this is the makeup of those groups. Further, although cluster analysis has identified two groups, it does not mean that one group is truly sophisticated and the other truly naive. We must do additional testing to determine how well each group fits our definition of naive or sophisticated.

Classifying the Asymmetric Auction Subjects We take a different approach to classifying the subjects in the asymmetric auctions. Rather than estimating individual bid functions and comparing them to naive and sophisticated bid functions, we use a strict criteria based on winning and losing auctions when advantaged or regular. We made this change for three reasons. First, the sophisticated advantaged bid function is distinctly different from the sophisticated regular bid function. This means we would need to run two separate regressions for each individual; one for advantaged play and one for regular play.

Second, we have 17 to 20 observations for each subject. But, subjects were advantaged or regular an equal number of time. The number of observations for advantage bidding ranges from 4 to 13 for an individual subject.

Finally, advantaged players won 75.6% of all the auctions. This mean that most of the observations for advantaged players are censored. The need for two separate regressions, the small number of observations, and the censoring of winning bids all combine to make it extremely difficult to estimate bid functions for comparison with naive and sophisticated bid functions. Fortunately, sophisticated players are expected to win all auctions when advantaged and lose all auctions when regular. We used this as our criteria for classifying the subjects.

We first looked for subjects that met the strict criteria of winning all auctions when advantaged and losing all auctions when regular. No subjects met this criteria. We relaxed the criteria to require that subjects win only 90% of the auctions when advantaged and win no more than 10% of the auctions when regular. Two subjects or 6% satisfied this criteria. We relaxed the criteria further and defined the 80% criteria as winning 80% of the auctions when advantaged and winning no more than 20% of the auctions when regular. Seven subjects or 21% met this criteria.

We relaxed the criteria one more time to winning 70% when advantaged and winning no more than 30% when regular. Seventeen subjects or 51% met the criteria.[9] This is summarized in Table 8.

[INSERT TABLE 8 ABOUT HERE]

The 80% criteria is not very strong. If players strictly followed the naive (expected value) bid function, then given the actual signals realized in the experiment, advantaged players should have won 85.7% of the auctions. We found it surprising that so few subjects met the 90% or 80% criteria. We expected to see more subjects classified as sophisticated in the asymmetric auctions that in the symmetric auctions. Achieving the correct bid function in the symmetric auction requires understanding how to calculate what the expected value is conditional upon winning. By contrast, the asymmetric problem appears much easier to solve; win when advantaged, lose when regular. We also expected that using such a large advantage would reinforce this idea. It is difficult to win an auction when regular in our design and earn positive profits. Players that won when regular lost an average of \$0.17 per auction, while advantaged winners averaged profits of \$1.80 per auction. This did not seem to affect players behavior in the auctions as the percentage of auctions won by regular bidders remained stable over the last ten rounds. Winning regular bidders did not appear to improve over time by losing less money. Average losses for winning regular bidders over the last ten rounds were \$0.34 per auction.

7.0.7 Comparing Naive & Sophisticated Bidders

Once the subjects in the symmetric and asymmetric auctions were classified, we tested to see if the groups are truly different, and if they differ in the ways predicted by our model.

Symmetric Auctions First, we specified a variable threshold Tobit model with random effects for each type as follows:

$$B_{it} = \alpha_0 + \alpha_1 X_{it} + \epsilon_{it} \tag{22}$$

[9] We did not want to punish individuals for winning or losing an auction because of highly irrational behavior by an opponent. For example, sophisticated regular bidders should stay in the auction until the clock reaches $X + 1$. This is the minimum possible value to the regular bidder. But, suppose the advantaged opponent dropped before the clock reached X, the regular bidder's signal. The sophisticated regular bidder was following the correct bid function, but lost the auction because the advantaged opponent behaved in a highly irrational way. Likewise, an advantaged bidder could lose an auction because the regular opponent stayed in the auction beyond any reasonble point. So, we looked for such situations. We found three auctions where a regular bidder won because the advantaged bidder dropped before the clock reached the minimum possible value to the regular bidder. Two cases affected our classification of bidders. In one case, a bidder that would have been classified as sophisticated by the 80% criteria is now classified as sophisticated by the 90% criteria. In the second case, a bidder that would be sophisticated by the 70% criteria is now sophisticated by the 80% criteria. The third case had no effect on our classification. We also found seven cases where the regular bidder stayed in the auction beyond the maximum possible value to the regular bidder, but the advantaged players won the auctions in all seven cases. Thus, these cases had no effect on our classification of subjects.

and tested each group against the expected value and Nash equilibrium models. The results are shown in Table 9. We reject both models for both groups, but the sophisticated group does come closer to the expected value model. It also appears when we nexamine the differences between all the auctions and the last 10 rounds that the sophisticated players are learning over time, while the naive players are not.

The sophisticated player's bid function is not linear in Region II of Figure 2. So, we also tested the sophisticated group by dropping observations from Region II and running the regression only on Regions I & III and comparing the results to the Nash equilibrium predictions. These results are shown in Table 10. Again, we must reject the Nash equilibrium bidding model.

[INSERT TABLES 9 & 10 ABOUT HERE]

Our naive/sophisticated model predicts that naive players bid too aggressively relative to the Nash equilibrium for signals low signals, and not aggressively enough for high signals. We set $\lambda = .7$, based on our classification, and calculated predicted wins and losses using the sophisticated nbid function against a naive player. Table 11 shows the actual percentage of auctions wons for each type as well as the predicted percentage won for naive or sophisticated players. Note that in all regions, the sophisticated players are much closer to the naive predictions than the sophisticated predictions. The sophisticated players are also closer to the naive predictions than the naive group.

[INSERT TABLE 11 ABOUT HERE]

Thus, while we cannot say that the sophisticated group is truly sophisticated, we can say that they are more sophisticated than their naive counterparts. The players classified as sophisticated seem to understand that they should bid aggressively when they have a high signal, but they do not seem to understand that they should bid cautiously when they have a low signal.

Asymmetric Auctions We tested the asymmetric auctions using the following specification.

$$B_{it} = \alpha_0 + \alpha_1 X_{it} + \alpha_2 C_i + \epsilon_{it} \tag{23}$$

Again, $C_i = 1$ for the sophisticated players and 0 otherwise. Because sophisticated players are expected to behave very differently when advantaged than when regular, we ran separate regressions for advantaged players and regular players. We ran the regression using the classifications from the 80% criteria (7 subjects classified as sophisticated and 26 as naive). We did not include an interaction term as the slope of the predicted bid functions for both types of players is one. The classification dummy should have a positive sign for the advantaged players and a negative sign for the regular players. These results are shown in Table 12. The classification dummy has the correct sign and is significant over all the rounds and over the last 10 rounds for the advantaged bidders. However, the classification is not statistically significant for the regular bidders over all the rounds or over the last 10 rounds.

[INSERT TABLE 12 ABOUT HERE]

We also tested the effect of the advantage within each type of players. We specified:

$$B_{it} = \alpha_0 + \alpha_1 X_{it} + \alpha_2 D_{it} + \epsilon_{it}$$

where $D_{it} = 1$ if the player is advantaged and 0 otherwise. These results are shown in Table 13. As expected from our classification criteria, the sophisticated players respond much more strongly to the advantage than the naive players. We tested the bid functions of each group against the naive and sophisticated bid functions. Interestingly, we must reject both bid functions for the naive players, but we cannot reject the naive bid function for the sophisticated players.

These results indicate that players classified as sophisticated bid more aggressively than their naive counterparts when advantaged, but do not reduce their bids (relative to naive players) when regular. Sophisticated players should understand that they should win when advantaged even with a low signal, and should lose when regular even with a high signal. Table 14 shows the actual percentage of auctions won as well as the naive and sophisticated predictions for the three regions of signals noted in Figure 2. If a sophisticated player truly understands the implications of an advantage, they should not win when regular in Region III and should not lose when advantaged in Region I. As shown in Table 14, the sophisticated players actually come closer to the naive predictions than the sophisticated predictions. Clearly, they do not understand that they should lose when regular even with a high signal. However, given that the sophisticated players only satisfy the very weak 80% criteria, it is not surprising that we see little effect in the bid functions or in the win/loss criteria in Regions I & III.

[INSERT TABLES 13 & 14 ABOUT HERE]

The sophisticated players appear to understand that they should be aggressive when they have an advantage, but they do not appear to understand that they should be cautious when regular.

Comparing the Groups Across Auctions As a last test of our classifications, we compared the behavior of bidders in the symmetric and asymmetric auctions by classification. We specified a variable threshold Tobit with random effects:

$$B_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 T_{it} + \epsilon_{it} \tag{24}$$

where $T_{it} = 1$ for the asymmetric auctions and 0 otherwise. The results are shown in Table 15.

[INSERT TABLE 15 ABOUT HERE]

We compared the naive regular bidders in the asymmetric auctions with the naive bidders in the symmetric auctions. The treatment dummy variable is significant but small over all the rounds. Over the last 10 rounds, the treatment dummy is small and statistically insignificant. This is consistent with the predictions of our model. Naive regular players do not alter their bidding in the asymmetric auctions relative to the symmetric auctions. We also compared the sophisticated regular bidders of the asymmetric auctions to the sophisticated bidders in the symmetric auctions. Here, the effect of the treatment dummy is insignificant for all the rounds and over the last 10 rounds. This is inconsistent with our model, which predicts that regular sophisticated players bid significantly lower in the asymmetric auctions compared to the symmetric auctions. It is however, consistent with our earlier results showing that the classification dummy was not significant for the regular bidders in the asymmetric auctions. It is also consistent with our results from the symmetric auctions showing little difference in the percentages of auctions won by naive and sophisticated players for low signal values.

8 Discussion and Conclusions

We do not find evidence of the explosive effect in our experiment. The majority of bidders in both the symmetric and asymmetric auctions use an expected value bid function. As a result, the effect of an advantage on the outcome and revenue is proportional rather than explosive. The effect on revenue is small. However, the expected value model does not explain all the data.

We have developed a behavioral model with two types of players that exploits the heterogeneity of the bidders and provides another possible explanation for the lack of an explosive effect in the asymmetric auctions. In our model, the effect of a 'small' advantage ($K < \frac{1}{2}$) is expected to be proportional to the size of K. It is only when $K \geq \frac{1}{2}$, that the explosive effect is restored. Our model defines players in the symmetric auctions as naive or sophisticated based on their bidding behavior. Using cluster analysis, we classified bidders in the symmetric auctions based on how closely they followed the naive or sophisticated bid functions. We classified 10 or 30% of the subjects as sophisticated in the symmetric auctions.

We used a strict win/lose criteria for classifying the subjects in the asymmetric auctions. Sophisticated players should win all auctions in which they are advantaged and lose all auctions in which they are regular. No subjects met the strict criteria of winning 100% of advantaged auctions and winning 0% of regular auctions. We relaxed our criteria to 90% and 2 subjects met the criteria. At the 80% criteria, 7 subjects or 21% of the subjects met the criteria.

We were surprised by how few subjects met the 90% and 80% criteria. If players had strictly followed the expected value bid function, then advantaged players should have won 85.7% of the auctions. The 80% criteria is clearly below this figure. We used a large value of K, equal to $\frac{1}{2}$ on the $[0, 1]$ interval to restore the explosive effect in the asymmetrical auctions. We expected this to simplify the problem for the subjects. Rather than calculating the expected value of the item conditional on winning, they simply needed to win when advantaged and lose when regular.

After we had classified the players in both symmetric and asymmetric case, we ran tests to determine if the groups really were naive or sophisticated. Our model predicts that in the symmetric auctions naive players will win more frequently for low signals than the sophisticated players, and naive players will lose more frequently than sophisticated players for high signal values. While sophisticated players do win more frequently than naive players for high signal values, there is no difference between the sophisticated players and the naive predictions. In the asymmetric auctions, our model predicts that sophisticated players will be more aggressive (bid higher) than naive players when advantaged, but will be less aggressive (bid lower) than naive players when regular. We found that the sophisticated players did bid higher for any given signal than the naive players when advantaged, but did not lower their bids relative to the naive players when regular. The model also predicts that sophisticated players should lower their bids relative to the symmetric auctions when regular, but they do not.

We have identified two groups of players in both the symmetric and asymmetric auctions. The group classified as sophisticated bids more aggressively for high signals and when advantaged than do their naive counterparts. However, the sophisticated group does not reduce their bids relative to the naive players for low signals or when regular. The sophisticated players are more sophisticated than their naive counterparts in terms of responding to higher signals and an advantaged, but they are not very sophisticated as they do not respond by lowering their bids over low signals or when regular.

A Appendix

In Part I of this appendix we present the proof of Proposition 1. Part II presents the proof of Proposition 2.

PART I

In the symmetric case, our model has two type of bidders: Naive and Sophisticated, denoted by N, and S, and let $B_N(x)$ and $B_S(x)$ respectively, denote their bidding functions. As motivated in the text, $B_N(x) = \frac{1}{2} + x$. Assume that there exists an interval of bids, $[\underline{B}, \overline{B}]$, such that if $B \in [\underline{B}, \overline{B}]$, then it could have come from a bid by either the N bidder or the S bidder. In other words, $\forall B \in [\underline{B}, \overline{B}]$, there exists x and y such that $B_N(x) = B_S(y) = B$. Having such a range, $[\underline{B}, \overline{B}]$, in equilibrium of a second-price-auction, implies that the S bidder must be indifferent, in the event of a tie, between winning and paying that (tied) bid or losing (the tie breaker). We now derive such equilibrium conditions. Consider a S bidder who has a signal x and considers bidding $B \in [\underline{B}, \overline{B}]$. If such a bid yields a tie with the N bidder, it implies $y = B - \frac{1}{2}$, where y is the signal of the N bidder. Alternatively, if such a bid yields a tie with a rival S bidder, it implies, in a symmetric equilibrium, that the rival's signal is also x. Thus the indifference equilibrium condition is:

$$B_S(x) = x + \delta(x)[B_S(x) - \frac{1}{2}] + (1 - \delta(x))x, \tag{25}$$

where, conditional on $B_S(x)$ yielding a tie, $\delta(x)$ is the probability that the tieing bid comes from a bid by a N rival and $(1 - \delta(x))$ is the probability that the tieing bid comes from a S rival. Since we are interested in probabilities that are conditional on the event of a tie, the *prior* probability of facing each rival, λ and $(1 - \lambda)$, are not the relevant probabilities here as we must use the *posterior* probabilities. Below we derive $\delta(x)$ but first we make a few simple observations. It is easy to see that equation 25 can be simplified (we do it for $\delta(x) < 1$, as otherwise there cannot be a tie with the S rival.):

$$B_S(x) = 2x + \frac{\delta(x)}{(1 - \delta(x))})[x - \frac{1}{2}], \tag{26}$$

Note that **any** solution to equation 26 implies $B_S(\frac{1}{2}) = 1$, as we have in our Figure 2.

$\forall B \in [\underline{B}, \overline{B})$, consider $\varepsilon > 0$, small enough so that $[B, B - \varepsilon] \subset [\underline{B}, \overline{B})$ and consider

$$\text{Pr[Rival's bid belongs to } N | \text{Rival's Bid } \in [B, B - \varepsilon]] =$$

$$\frac{\text{Pr[Rival's Bid } \in [B, B-\varepsilon] | \text{Rival's bid belongs to } N]Pr\{\text{Rival is } N\}}{Pr[\text{Rival's bid } \in [B, B-\varepsilon] \mid \text{Rival's bid belongs to } N\}Pr\{\text{Rival is } N\} + Pr[\text{Rival's bid } \in [B, B-\varepsilon] \mid \text{Rival's bid belo}}$$

$$\frac{Pr[\text{Rival's bid } \in [B, B-\varepsilon] \mid \text{Rival's bid belongs to } N\}\lambda}{Pr[\text{Rival's bid } \in [B, B-\varepsilon] \mid \text{Rival's bid belongs to } N\}\lambda + Pr[\text{Rival's bid } \in [B, B-\varepsilon] \mid \text{Rival's bid belongs to } S\}(1 -}$$

Given that signals are distributed uniformly on $[0, 1]$, and that the N strategy is $B_N(x) = \frac{1}{2} + x$, we can simplify and Rewrite the above expression as:

$$Pr[Rival\prime sbidbelongstoN|Rival\prime sbid \in [B, B-\varepsilon]\} = \frac{\varepsilon\lambda}{\varepsilon\lambda + (1-\lambda)[\sigma(B) - \sigma(B-\varepsilon)]} = \frac{\lambda}{\lambda + (1-\lambda)[\sigma(B) - \sigma(B-\varepsilon)]/\varepsilon} \tag{27}$$

However since, $\lim\limits_{\varepsilon\to 0} [\sigma(B) - \sigma(B-\varepsilon)]/\varepsilon = \sigma'(B) = 1/B_S'(x)$, we conclude that $\delta(x) = \frac{\lambda B_S'(x)}{\lambda B_S'(x) + (1-\lambda))}$ and that

$\frac{\delta(x)}{(1-\delta(x))}) = \frac{\lambda B_S'(x)}{(1-\lambda)}$. Substituting in equation 26, we finally derive equation 6 in the text:

$$B_S(x) = 2x + \frac{\lambda B_S'(x)}{(1-\lambda)}[x - \tfrac{1}{2}]$$

I still have to say here (relative easy) about initial conditions. $B_S(\frac{1}{4}) = \frac{1}{2}$; $B_S(\frac{3}{4}) = \frac{3}{2}$.

PART II

In this section we present the proof of Proposition 2.

The asymmetric case for a small advantage, that is, $K \in (0, \frac{1}{2})$, is more complicated to derive and solve than in the symmetric case. However, in our experimental design we wanted to have a "large" K. Whenever $K \geq \frac{1}{2}$ the asymmetries are large enough that we can solve for equilibrium directly. In the asymmetric case we have four types of bidders: Naive-Regular, Naive-Advantaged, Sophisticated-Regular and Sophisticated-Advantaged, denoted by N^R, N^A, S^R, and S^A respectively. The bidding strategy for the two naive types is simple as the bid functions are by assumption. Let $B_N^R(x)$ and $B_N^A(x)$ denote the bidding functions for the two naive types. They are given by:

$$B_N^R(x) = \frac{1}{2} + x, \qquad B_N^A(x) = \frac{1}{2} + K + x. \tag{28}$$

Let $B_S^R(x)$ and $B_S^A(x)$ denote the bidding functions for the two sophisticated types, Regular and Advantaged and assume that they are strictly monotonic (which will be verified in equilibrium). Let $\sigma_S^R(B)$ and $\sigma_S^A(B)$ denote the inverse of these two functions respectively. We first assume that there exists an interval of bids, $[\underline{B}, \overline{B}]$ such that if $B \in [\underline{B}, \overline{B}]$, then it could have come from S^R or S^A. (Recall that it is a common-knowledge who is a Regular bidder and who is the Advantaged bidder.) In other words, $\forall B \in [\underline{B}, \overline{B}]$, there exists x and y such that, $B_S^R(x) = B_S^A(y) = B$. But how could such an interval, $[\underline{B}, \overline{B}]$, exist with $K > 0$? As in Levin & Kagel, 2004, we may escape the "explosiveness" result of the two bidders almost-common-value literature due to the existence of a N ^{R} bidder who is aggressive enough to discipline the S^A bidder This discipline reduces the adverse selection problem for the S^R bidder who may now (in equilibrium while guarding against the winner's curse) attempt to win and thus allowing for such a $[\underline{B}, \overline{B}]$.

Having such a range, $[\underline{B},\overline{B}]$, in equilibrium of a second-price-auction, implies that both S^R and S^A must be indifferent, in the event of a tie, between winning and paying that (tied) bid or losing (the tie breaker). We now derive such equilibrium conditions. Consider a S^R (S^A) bidder who has a signal x and consider bidding $B \in [\underline{B},\overline{B}]$. If the tie comes with N^A (N^R) bidder, the rival's signal, y, must be $y = B_S^R(x) - \frac{1}{2} - K$, ($y = B_S^A(x) - \frac{1}{2}$). Alternatively, if the tie comes with a S^A (S^R) bidder the rival's signal, y, must be $y = \sigma_S^A(B_S^R(x))$, ($\sigma_S^R(B_S^A(y))$). Thus, we can write:

$$B_S^R(x) = x + \delta(x)[B_S^R(x) - \frac{1}{2} - K] + (1 - \delta(x))\sigma_S^A(B_S^R(x)), \tag{29}$$

and

$$B_S^A(y) = y + K + \widetilde{\delta}(y)[B_S^A(x) - \frac{1}{2}] + (1 - \widetilde{\delta}(y))\sigma_S^R(B_S^A(y)), \tag{30}$$

where $\delta(x)$ is the probability that conditional on $B_S^R(x)$ yielding a tie, the tieing bid comes from a bid of the N^A.and where $\widetilde{\delta}(y)$ is the probability that conditional on $B_S^A(y)$ yielding a tie, the tieing bid comes from a bid of the N^R. As in the symmetric case here both $\delta(x)$ and $\widetilde{\delta}(y)$ involve computing *posteriors*, using Bayes rule, and thus, as before, the expressions $\delta(x)$ and $\widetilde{\delta}(y)$ contain $\frac{dB_S^R(x)}{dx}$ and $\frac{dB_S^A(y)}{dy}$ respectively.[10]

With simple algebra these two equations can be simplified to:

$$B_S^R(x) = \sigma_S^A(B_S^R(x)) + x + \frac{\delta(x)}{1 - \delta(x)}[x - (K + \frac{1}{2})], \tag{31}$$

and,

$$B_S^A(y) = \sigma_S^R(B_S^A(y)) + y + K + \frac{\widetilde{\delta}(y)}{1 - \widetilde{\delta}(y)}[y + K - \frac{1}{2}]. \tag{32}$$

].

Assume momentarily that there exists $y \in [0,1]$ and $z \in [0,1]$, such that, $B_S^R(y) = B_S^A(z)$. It immediately follows from equation 31 and equation 32 that in such a case,

$$B_S^R(y) = \sigma_S^A(B_S^R(y)) + y + \frac{\delta(y)}{1 - \delta(y)}[y - (K + \frac{1}{2})] = z + y + \frac{\delta(y)}{1 - \delta(y)}[y - (K + \frac{1}{2})] \tag{33}$$

and

[10] $\delta(x)$ and $\widetilde{\delta}(x)$ are derived in the same way we derived the in the symmetric case. Since we solve directly for the equilibrium for our specific design further details, including derivation of initial conditions are omitted.

$$B_S^A(z) = \sigma_S^R(B_S^A(z)) + z + K + \frac{\widetilde{\delta}(z)}{1-\widetilde{\delta}(z)}[z + K - \frac{1}{2}] = y + z + K + \frac{\widetilde{\delta}(z)}{1-\widetilde{\delta}(z)}[z + K - \frac{1}{2}] \quad (34)$$

and therefore $\frac{\delta(y)}{1-\delta(y)}[y - (K + \frac{1}{2})] = K + \frac{\widetilde{\delta}(z)}{1-\widetilde{\delta}(z)}[z + K - \frac{1}{2}]$.

However, for $K \geq \frac{1}{2}$ that leads to a contradictions as $\frac{\delta(y)}{1-\delta(y)}[y - (K + \frac{1}{2})] \leq 0 < \frac{1}{2} \leq K + \frac{\widetilde{\delta}(z)}{1-\widetilde{\delta}(z)}[z + K - \frac{1}{2}]$.

Thus, $K \geq \frac{1}{2}$ is sufficiently large to rule out a common bid range, $[\underline{B}, \overline{B}]$, for S^A and S^R. Next we show that with $K \geq \frac{1}{2}$, the S^R does not wish to win against a N^A rival: If x is the signal of S^R and signal y is the rival's, N^A, signal, then upon winning ,S^R's value is $(x + y)$ and pays $B_N^A(y) = K + \frac{1}{2} + y$, for an earning of $(x - K - \frac{1}{2}) < 0$. Thus, in equilibrium S^R uses the least aggressive bidding strategy that is not dominated: $B_S^R(x) = x$. (Since with a signal of x and a value of $x + y > x$, any bid $B_N^R(x) < x$ is weakly dominated by $B_N^R(x) = x$.) Given the equilibrium bidding strategies specified, it is easy to show that S^A always wishes to win: With a signal of x and rival's signal of y, S^A's value upon winning is $(K + x + y)$ and she pays either y, if S^R sets the price, and it is $y + \frac{1}{2}$, if N^R sets the price. In both instances S^A makes strictly positive payoffs. To assure winning, S^A must at least bid $\frac{3}{2}$, but may bid higher. However, since the value for a S^A never exceeds $(\frac{1}{2} + x + y) \leq (\frac{3}{2} + x)$, $B_S^A(x) > (\frac{3}{2} + x)$ is dominated by $B_S^A(x) = (\frac{3}{2} + x)$.

We summarize now the equilibrium bidding for all four types of bidders in our asymmetric case with $K = \frac{1}{2}$:

$$B_N^R(x) = \tfrac{1}{2} + x \qquad B_N^A(x) = 1 + x \qquad B_S^R(x) = x; \qquad \tfrac{3}{2} \leq B_S^A(x) \leq (\tfrac{3}{2} + x).$$

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