On the Job Search and the Wage Distribution

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Abstract

Estimates of the structural parameters of a job separation model derived from the theory of on-the-job search are reported in this paper. Given that each employer pays the same wage to observably equivalent workers but wages are dispersed across employers, the theory implies that an employer’s separation flow is the sum of an exogenous outflow unrelated to the wage paid and a job-to-job flow that decreases with the employer’s wage. The specification estimated allows worker search effort to depend on the wage currently earned. The empirical results imply that search effort declines with the wage paid, as the theory predicts, using Danish IDA data for the years 1994-1995. Furthermore, the estimates for the full sample and four occupational sub-samples explain the employment effect, defined as the horizontal difference between the distribution of wages earned and the distribution of wages offered.

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1 Introduction

Ample evidence suggests that employers pay observably similar workers different wages.\(^1\) Two explanations are offered in the literature: either employers pursue different wage policies and/or high wage firms attract more able workers.\(^2\) Recent empirical studies by Abowd and Kramarz (2000a, 2000b), based on the analysis of matched employer-worker data for both the U.S. and France, conclude that the two are equally important as explanations of inter-industry differentials and that wage policy differences explain 70% of the size differentials.

It is surprising that so little is known about actual firm wage policies, other than that wage differences for observationally equivalent workers exist. Human resources textbooks, such as Milkovitch and Newman’s *Compensation*, discuss many aspects of wages but provide no suggestions about what wage policy should be. Even the personnel economics literature, for example Eddie Lazear’s *Personnel Economics for Managers* or Baron and Kreps’ *Strategic Human Resources*, has omitted discussion of optimal wage policy. This omission is surprising because the essential elements of a theory of wage policy have appeared in Samuelson’s principles of economics textbook since 1951. Samuelson writes:

**Wage policy of firms.** The fact that a firm of any size must have a wage policy is additional evidence of labor market imperfections.... But just because competition is not 100 per cent perfect does not mean that it must be zero. The world is a blend of (1) competition and (2) some degree of monopoly power over the wage to be paid. A firm that tries to set its wage too low will soon learn this. At first nothing much need happen; but eventually it will find its workers quitting a little more rapidly than


\(^2\)Krueger and Summers (1988) emphasized the former explanation, while Murphy and Topel (1987) argued that unmeasured differences in individual ability are the principal explanation. Although work by Dickens and Katz (1987) and Gibbons and Katz (1992) attempted to resolve the debate, their efforts and those of others were hampered by lack of appropriate matched worker-employer data.
would otherwise be the case. Recruitment of new people of the same quality will get harder and harder... Availability of labor supply does, therefore, affect the wage you set under realistic conditions of imperfect competition. [p.554] ³

To the extent that wage policies differ, the typical worker has an incentive to seek out higher paying firms as suggested in Samuelson’s comments. Indeed, on-the-job search motivated by wage dispersion provides an explanation for the commonly observed negative association between wages paid and separation flows in a cross-section of firms.⁴ The theory also implies that the wage earned increases in the stochastic sense with the elapsed duration since the worker’s last non-employment spell as a consequence of job-to-job movement. This implied employment effect on the wage earned provides another interpretation of positive tenure and experience coefficients in empirical wage equations. Determining whether an employment effect exists and documenting that its magnitude can be explained by a simple on-the-job search model is a major contribution of this paper.

The principal task of this paper is to estimate a structural model of worker separations based on the theory of on-the-job search using cross-firm observations on separation flows and to test the associated implications of the theory for the differences between the distribution of wages offered and the distribution of wages earned. Burdett (1978) provides the original formal treatment of search on-the-job given wage dispersion across employers. In his model, employers pursue a stationary wage policy by assumption, an unemployed worker accepts the first offer received above some reservation wage, and an employed worker moves to a higher paying job when the opportunity arises. Mortensen (1990) demonstrates that the process by which workers move from one job to another will generate a distribution of wages earned over employed workers which stochastically dominates the distribution of wages offered applicants.

³Samuelson’s text adds and deletes information in each version. The material quoted here is not in the 1948 edition, appearing first in 1951 and remaining intact through the 1989 edition.

⁴For a review of this literature, see Farber (1999).
The location difference between the two distributions, here called the employment effect, is a consequence of the fact that employed workers move up the “job ladder” by flowing from lower to higher paying jobs without intervening spells of non-employment. The formal model used in the estimation is a generalization of Burdett’s theory that allows for an endogenously chosen search intensity. The data strongly supports the need for incorporating the choice of search effort into the model. To reach this conclusion we have to make strong assumptions about structure, which we do. Job destruction rates are assumed exogenous and common across employers; workers are regarded as homogenous so that we can meaningfully compute a firm’s wage. Some of these assumptions could be relaxed, but there is a “no free lunch” theorem lurking in the background: one either believes in a search model or in a firm-specific human capital story to interpret wage data. For reasons that we detail below we pursue the search frictions approach. There is as yet no theoretical underpinning for a firm-specific capital cum search model\textsuperscript{5}.

The data used in the estimation are based on the Danish Integrated Database for Labour Market Research (IDA). This matched employer-employee data source, a product of Statistics Denmark, includes employment and wages paid on an annual basis as well as employee characteristics including employment status in the previous year in all workplaces in Denmark since 1980. The data of interest for this paper include cross-section information on the total number of workers employed in each firm in November of 1994, the number of these who are still employed one year later, and the hourly wage paid each employee during the survey year, November 1994 to November 1995. Information on the occupation membership of each employee is also available in the data set and is used in this paper to create the sub-samples studied. The occupations include managers, salaried workers, skilled workers and unskilled workers.

\textsuperscript{5}Postal-Vinay and Robin (2002) provide a search and bargaining story where wages increase on a job because of outside offers, but there is no human capital accumulation in this model.
To focus on the cross-firm distribution of wages we define an employer’s wage as the average hourly wage paid to its employees. Our focus on average firm wages is uncommon; it is based on three observations. First, given any search theory of job-to-job movements based on firm wage differentials it is only the firm component that matters: differences in personal ability simply confuse the issue. And job-to-job movements are quantitatively important. Peter Matilla (1974) was the first to note that between 50-60% of job transitions did not involve a spell of unemployment; Bowlus et al. (2001) report that 44% of the job-transitions of younger males in the NLSY79 data are direct job-to-job moves. Second, under the identifying assumption that worker and firm components of the wage are independent, firm averages allow us to abstract from irrelevant differences in ability. In other words, under this assumption differences in average wages equal differences in firm components plus noise. Independence in worker-firm components holds in other data sets (see Abowd, Kramarz and Margolis (1999) for the case of France) and could be tested using the Danish data, a task we leave for future work. We also note that in these data the cross-firm variance in (log) wages accounts for 60 to 70% of the total variation in wages; in other words, the lion’s share. Third, the approach we use ignores the effect of tenure on wages in order to focus on equilibrium relations. This is not unreasonable because the effect of tenure on wages is generally agreed to be small. Altonji and Williams (1997) place the consensus tenure effect at between 6.6% and 11% per decade.\textsuperscript{6} This is a small part of the average wage growth that occurs in a decade. For example, in Census data for 1970 and 1980 earnings of males aged 31-35 were 87% greater than earnings of males aged 21-25. In 1990 and 2000 the differential was 109%.\textsuperscript{7} Even assuming that everyone worked the entire 10 years at the same employer and that the Altonji-Williams estimate of 11% is correct, tenure effects account for only 10% (= 11/109)


\textsuperscript{7}The Census data are from the IPUMS project and are available on the web at http:\\www.ipums.mn.edu.
to 13% (= 11/87) of average wage growth over a decade. The same pattern occurs if we look at males aged 41-45 and compare them to males 21-25. Of course, these groups are not identical: older workers have more education, but that differs by less than 1/10th of a year. Because most workers separate from their employer of ten years ago, these tenure effects are overstated.

In our view, it makes more sense to focus on the 87-90% of wage growth that is not explained by firm-specific human capital models. Because we ignore tenure effects, the employment effect that we discuss in section 4 below is then upward biased but, as the previous calculations indicate, the bias is not likely to be large. We comment further on this point in section 4.

The distribution of wages earned is the employment size weighted distribution of employer wages while the distribution of wages offered is weighted by the relative number of workers hired by each firm from non-employment. Because the data source matches employment and earnings histories of individual workers with their employing firms, both distributions are observed in these data. The employer separation function is estimated under the maintained assumption that all workers in the specified sub-sample under study are equally productive in every firm. In other words, the maintained hypothesis is that cross firm differences in the average hourly wage paid represent pure wage dispersion attributable to heterogeneity in wage policies. The results are reported for sub-samples defined by worker occupation as well as for the total sample.

The estimates of the separation model parameters imply a strong negative relationship between search effort and wage for all occupations. In other words, search intensity is high for workers employed in low wage jobs but drops off, typically quite dramatically, as the wage earned by an employed worker increases and tends to zero as the wage earned tends to the highest paid. Because workers who currently earn less have more to gain by searching more
intensively, these results support the theory of optimal on-the-job search effort. An estimate of the curvature parameter of the cost of search function is identified, in spite of the fact that search effort is not itself directly observed. Although the parameter estimates vary across occupations, the result for the full sample suggests that a quadratic cost of search effort is a good approximation. We note that although we use a specific functional form for the cost of search function, namely a member of the power law family, the curvature is identified, up to a scaling constant, non-parametrically. This is discussed in section 3.1.

Given the model’s implications for employment and wage mobility, the distribution of wages earned by employed workers obeys a law of motion that depends only on the wage offer distribution and the separation function. Hence, the estimated separation function and observed offer distribution can be used to solve for a theoretical steady state distribution of wages earned by employed workers. The implied theoretical distribution can be compared with the actual distribution of earned wages found in the data. Indeed, doing so provides an independent test of the theory since the observed distribution of wages earned is not used to estimate the model. As predicted by the theory, the actual distribution of wages earned in each of our data sets always lies to the right of the distribution of wages offered. Furthermore, the observed distribution of wages earned and that predicted by the estimated model are remarkably close for both the full sample and the four occupational sub-samples studied in the paper. Hence, the model passes this rather stringent ‘out of sample’ test. It may be noted that other theories of wage formation, e.g., firm-specific human capital, predict a difference between the offer and earnings distributions. However, these theories do not imply the rates of turnover seen in the data. For example, total separations average 30% of employment over the years 1981 to 1996. Workers with less than 1 year of tenure turned over at the rate of 50%, while workers with 5 years of tenure separated at a rate of 18%. Indeed, the tenure-specific turnover rate in these data never goes below 12% per year.
Turnover rates of this magnitude clearly indicate the importance of on-the-job search.

Closely related papers are few. Other than work that documents the fact that job-to-job flows are relatively large, we are aware of only a few attempts to estimate a structural model of these flows at the micro level. Among recent examples, Bontemps et. al (2000) and Rosholm and Svarer (1999) estimate an empirical competing hazard job separation model using panel data on worker job histories. Although a new job is one of the destination states in their analyses, they implicitly assume that search effort is independent of the worker’s current wage. Yashiv (2000) estimates the parameters of a search effort cost function, as we do, but his workers search only when not employed. Furthermore, his estimates are based on aggregate time-series data. Still, his preferred specification is a quadratic cost function, approximately like that estimated here for the complete sample.

The rest of the paper is laid out as follows. Section 2 presents the fundamental model of job separation estimated in the paper and derives the steady state wage distribution implied by it and the offer distribution. Section 3 introduces the maximum likelihood estimation procedure and the data set. Section 4 discusses the results for both the full sample and for the occupational sub-samples. Section 5 concludes.

2 Job Search and Wage Dispersion

2.1 A Model of Job Separation

The model is in the spirit of Burdett (1978). All workers are identical labor market participants. Each acts to maximize expected wealth and lives forever. Let $w$ represent an employed worker’s current wage and let $F(w)$ represent the probability that a randomly selected wage offer is no greater than $w$, where each employer’s weight implicitly reflects relative recruiting effort. In other words, $F(w)$ is the fraction of “vacancies” that offer wage $w$ or less. To simplify the derivations below, the wage offer distribution is regarded as continuous.
Each worker receives outside offers at a Poisson frequency $\lambda s$ where $s$ is a measure of the worker’s search effort. Each worker chooses search effort subject to a twice differentiable increasing convex cost function $c(s)$ such that total and marginal cost are zero at the origin, i.e., $c(0) = c'(0) = 0$. Finally, any existing job-worker match ends for exogenous reasons at the exponential job destruction rate $\delta$. Then, under the assumption that each worker acts to maximize expected wealth, the current wage contingent value of employment, $W(w)$, solves the continuous time Bellman equation

$$rW(w) = \max_{s \geq 0} \left\{ w - c(s) + \lambda s \int \left( \max[W(x), W(w)] - W(w) \right) dF(x) + \delta (U - W(w)) \right\},$$

(1)

where $U$ is the value of non-employed search.

The difference between wage and search cost on the right side of equation (1) is the worker’s net current income. The next term on the right side represents the expected capital gain associated with the possible arrival of an outside offer, given that the worker acts optimally by accepting jobs with higher value. The last term reflects the expected capital loss attributable to job destruction, the difference between the value of unemployment and the value of employment in the worker’s current job. Hence, the equation is an arbitrage condition which defines the asset value of being employed to be that which equates the riskless return on the asset value of the search while employed option to current net income plus expected capital gains and losses associated with the option. This relationship is a continuous time equivalent of the well known Bellman equation of dynamic programming. Indeed, because equation (1) can be rewritten as

$$W(w) = \max_{s \geq 0} \left\{ \frac{w - c(s) + \delta U + \lambda s \int \max[\max(W(x), W(w)) dF(x)]}{r + \delta + \lambda s} \right\}$$

---

There is no loss of generality in the linearity of the relationship. However, the implicit assumption that workers who do not make an effort receive no offers does have content.
and because the right hand side satisfies Blackwell’s sufficient conditions for a contraction on the space of differentiable and increasing real valued functions, the value function is the unique fixed point of the contraction map on that space (see Stokey and Lucas (1989)).

Because the solution $W(w)$ to (1) is increasing in $w$, an employed worker accepts any offer greater than her current wage. Indeed,

$$W''(w) = \frac{1}{r + \delta + \lambda s(w)[1 - F(w)]} > 0$$

by the envelope theorem, where $s(w)$ is the optimal search effort choice. From the first order condition for an interior solution, integration by parts, and the appropriate substitution for $W'(w)$, it follows that

$$c'(s(w)) = \lambda \int_{\bar{w}}^{w} [W(x) - W(w)] dF(x) = \lambda \int_{\bar{w}}^{w} W'(x)[1 - F(x)] dx$$

(2)

where $\bar{w}$ is the upper support of the wage offer distribution. In other words, the optimal search effort function is the unique particular solution to this integral equation. Optimal search effort, $s(w)$, is strictly decreasing and continuous in the wage earned by convexity of the cost of search function.

Consider the same worker when not employed. The value of non-employment solves the analogous asset pricing equation

$$rU = \max_{s \geq 0} \left\{ b - c(s) + \lambda s \int [\max (W(x), U) - U] dF(x) \right\},$$

(3)

where $b$ represents income forgone when employed, e.g., the unemployment benefit. The worker’s reservation wage, $R$, is the solution to

$$W(R) = U.$$
denoted as $s_0$, equals search effort when employed at the worker’s reservation wage and, consequently, the worker’s reservation wage is simply the unemployment compensation, i.e.,

$$s_0 = s(R)$$

(4)

and

$$R = b.$$  

(5)

In sum, the overall job duration hazard for any worker employed by an employer paying wage $w$ is

$$d(w) = \delta + \lambda s(w)[1 - F(w)],$$

(6)

where $s'(w) < 0$ and $s(\bar{w}) = 0$. Under the assumption that an employer pays all workers the same wage and the cost of search is the same for all workers, the function $d(w)$ also represents the employer’s separation rate.

### 2.2 The Steady State Wage Distribution

Given the wage offer distribution, $F(w)$, and the model of worker flows reviewed above, the distribution of wages across employed workers, denoted as $G(w)$, converges over time to a unique steady state distribution in a stationary environment. The separation theory above predicts that the wages of employed workers generally exceed the wages offered workers by employers in the sense that $G(w)$ stochastically dominates $F(w)$. The purpose of this section is to derive the formal relationship between the two distributions. Both distributions are observable in our data, and the resulting relationship is an important testable model implication.

Workers flow from unemployment to employment at rate $\lambda s_0[1 - F(R)]$, equal to the product of the offer arrival rate and the probability that a randomly generated offer exceeds
the reservation wage $R$. Workers flow from employment to unemployment at the exogenous rate $\delta$. Hence, if the total number of participants is fixed, then the steady state fraction not employed, $u$, balances these two flows, i.e., $u$ solves

$$
\frac{u}{1-u} = \frac{\delta}{\lambda s_0[1 - F(R)]} = \frac{\delta}{\lambda s_0}
$$

(7)
since $F(R) = 0$ in any equilibrium.

By analogous reasoning, the flow of non-employed workers who obtain a job paying $w$ or less is $s_0\lambda[F(w) - F(R)]u$. Because employed workers only flow from lower to higher paying jobs, this is the total flow into the set of employed worker paid wage $w$ or less. The flow out of this subset of employed workers, which has measure $(1-u)G(w)$, is the flow of those who lose their jobs, equal to $\delta G(w)(1-u)$, plus the flow of those who find jobs paying more than $w$. Since the rate at which workers search depends the current wage, the flow that finds a wage higher than $w$ is

$$
\lambda \int_{w}^{w} s(x)[1 - F(w)](1 - u)dG(x),
$$

where $x \in [w, w]$ represents a wage in the interval of interest and $(1-u)dG(z)$ is the measure of workers earning that wage. Hence, the steady state solution for the distribution function $G(w)$ solves the integral equation

$$
\delta G(w) + \lambda[1 - F(w)]\int_{w}^{w} s(x)dG(x) = \frac{\lambda s_0[F(w) - F(R)]u}{1-u} = \delta F(w),
$$

(8)
where the last equality is implied by $F(R) = 0$ and equation (7).

Equation (8) has qualitative implications of considerable interest for the predicted relationship between the distribution of wages offered to new employees and the distribution of wages paid to workers who are already employed. Namely,

$$
\frac{F(w) - G(w)}{1 - F(w)} = \frac{\lambda}{\delta} \int_{w}^{w} s(x)dG(x) > 0, \text{ for all } w \in (w, w)
$$

(9)
implies that the wages paid employed workers are higher than those offered to new hires in the sense that $G(w)$ stochastically dominates $F(w)$. The horizontal difference between
the two distribution functions can be interpreted as an employment premium or employment effect on the wage. It arises because some employed workers flow from lower to higher paying jobs without intervening periods of non-employment. Note that the premium declines with the job destruction rate but increases with the offer arrival parameter because workers return to unemployment more frequently as δ increases but move to higher paying jobs more rapidly as λ increases.

3 Estimating the Separation Function

3.1 Estimation Procedure

The purpose of this section is to formulate the procedure for estimating the separation process, equation (6), using cross employer wage offer and separation data and the observed wage offer distribution. The search intensity function is the unique solution of the functional equation

\[ s(w) = \phi \left( \int_w^\infty \frac{\lambda(1 - F(x))dx}{r + \delta + \lambda s(x)(1 - F(x))} \right) \]

by virtue of equation (2), where \( \phi(\cdot) \) is the inverse of the marginal cost function \( c'(\cdot) \). The estimates that follow assume a cost function of the form

\[ c(s) = \frac{c_0 s^{1 + \frac{1}{\gamma}}}{1 + \frac{1}{\gamma}}, \]

where \( c_0 > 0 \) is a scale parameter and \( 1+1/\gamma \) with \( \gamma > 0 \) (for strict convexity) is the elasticity of search cost with respect to effort. Thus, the search effort function is the solution to the functional equation

\[ s(w) = \left( \frac{1}{c_0} \int_w^\infty \frac{\lambda[1 - F(x)]dx}{r + \delta + \lambda s(x)[1 - F(x)]} \right)^{\gamma}. \tag{10} \]

As search effort is not directly observed, the two factors of the offer arrival rate \( \lambda s(w) \) cannot be separately identified. As a consequence, the scale parameter \( c_0 \) in the cost function
is not identified. Equation (10) can be expressed as

$$\lambda(w) = \alpha \left( \int_w^{\pi} \frac{[1 - F(x)]dx}{r + \delta + \lambda(x)[1 - F(x)]} \right)^\gamma,$$

with the definitions

$$\lambda(w) \equiv \lambda s(w), \quad \alpha \equiv \frac{\lambda^{1+\gamma}}{c_0^\gamma}. \quad (12)$$

Thus, the endogenous wage contingent arrival rate $\lambda(\cdot)$ solves a functional equation, and one parameter can be recovered by combining $\lambda$ and $c_0$ into $\alpha$, for identification purposes. The structural parameters actually estimated are the elements of the triple $(\delta, \gamma, \alpha)$. For the sake of interpretation, we report the transformed triple $(\delta, \gamma, \lambda)$, with $\lambda$ the value of the arrival rate given employment at the lowest wage, i.e., $\lambda = \lambda(w)$, and we represent the search intensity function as the arrival rate relative to that of the lowest paid workers, $s(w) = \lambda(w)/\lambda(w)$. This representation corresponds to an appropriate choice of units of search effort, or equivalently to an appropriate choice of the scale parameter $c_0$.

The IDA contains cross firm observations on the number of workers employed in November, 1994, their earnings during the subsequent year until November 1995, the number of original employees who remain employed in November of 1995, and the number of non-employed workers hired during the year. Let $w_i$ represent the average hourly wage paid by employer $i \in \{1, 2, \ldots, N\}$, let $n_i$ denote the number of employees and $x_i$ represent the number of “stayers”, defined as those who were initially employed and stayed on the whole year until the following November. The implications of the theory for the probability distribution of stayers in each firm conditional on the firm’s wage and size are used to form the likelihood function for these firm level data conditionally on the model’s unknown parameter vector $(\delta, \gamma, \alpha)$ and “market prices” represented by the interest rate $r$ and the offer distribution $F(w)$, which are observed.

As the duration of employment at firm $i$ is exponential with hazard rate $d_i$ for any worker under the assumption that all are identical, the probability that an initially employed worker
does not leave during the year is $p_i = e^{-d_i}$. As $x_i$ is the realized number of stayers out of the total possible, $x_i$ is binomial with probability of “success” $p_i$ and “sample size” $n_i$, i.e.,

$$\Pr(x_i = x|n_i, d_i) = \binom{n_i}{x} (e^{-d_i}x)(1 - e^{-d_i})^{n_i-x}. \quad (13)$$

Conditional on $r$ and $F$, estimates of the parameters $(\delta, \gamma, \alpha)$ are obtained by maximizing

$$\ln L(\delta, \gamma, \alpha) = \sum_{i=1}^{N} \left[ \ln \binom{n_i}{x_i} - d_i x_i + (n_i - x_i) \ln(1 - e^{-d_i}) \right], \quad (14)$$

where for each firm $d_i$ is given by the following rewrite of equation (6)

$$d_i = \delta + \lambda(w_i)[1 - F(w_i)] \quad (15)$$

and where the function $\lambda(w)$, which depends on $\alpha, \gamma$, and $\delta$, is the solution to equation (11). It is useful to note that the function $\lambda(w)$ is non-parametrically identified in (15) and hence in principle the solution for $\lambda$ obtained from (11) can be compared as long as $[1 - F(w)]$ is observed. This fact illustrates how we are able to compare a constant search effort specification to a variable search effort model. The chosen $\lambda(w)$ function has to match up the separation rate with the firm’s relative wage position, $[1 - F(w)]$.

There are three complications in the actual procedure used to obtain the estimates reported below. First, wages, new hires, and employment are observed for the firms in our sample. We use these data to form a sample analogue of the market offer distribution function $F(w)$ by weighting each firm’s wage by the relative number of workers hired by that firm from non-employment. Only hires from non-employment are included in forming the weights because the theory implies a sample selection problem for direct job-to-job hires. Namely, according to the theory, no employed worker who is offered a wage less than or equal to the one currently earned will be observed among the new hires. Hence, if all new hires were included, those coming from employment would contribute only relatively high wages, and the resulting distribution would be biased upward in the sense that it would
stochastically dominate the true sample distribution. Because all non-employed accept any offer above the common reservation wage and because all wages offered in the market by participating employers must be no less than this minimum, there is no selection problem for these workers.

Second, the interest rate \( r \) could be regarded as a parameter to be estimated. This is known to be difficult to do (Hall (1978), Campbell, Lo, and MacKinley (1997, Chapter 8)). We set the discount rate to the standard 5% per year\(^9\). Variation in this number between zero and 10% per year has no appreciable effect on the resulting estimates of the other parameters. Finally, the functional equation (11) does not yield a closed form solution for the search effort function \( \lambda(w) = \lambda s(w) \). Hence, at any likelihood function evaluation, \( \lambda(w) \) is solved numerically as a function of the underlying set of structural parameters by iterating on the mapping in (11) until an approximate fixed point is found. We evaluated the cdf \( F(w) \) at all integers between the minimum wage (69 DKK\(^10\)) and the maximum wage, a range typically of about 300 points depending upon the sub-sample used, and we solved for \( s(w) \) at each of the points. Convergence at iteration \( t \) was defined to occur when 

\[
\max_w |s_t(w) - s_{t-1}(w)| \leq 1.0 e^{-15}. 
\]

### 3.2 Data Description

The employers included in the IDA data are all privately owned Danish firms. Hence, the full sample is referred to as the private sector. Sub-samples are also constructed by stratifying the private sector sample by worker occupation. There are four exhaustive and mutually exclusive occupational categories: Skilled and unskilled workers, managers, and salaried workers. The firm observations are the average wage paid, the total number of employees in

\(^9\)We experimented with varying the (fixed) rate at levels up to 10% and found that the estimates were not sensitive to this variation. We also experimented with attempting to estimate \( r \) and found, as is common in the macro and finance literature, that it is difficult to obtain an precise and apriori sensible estimate.

\(^10\)There is no legal minimum wage in Denmark. The 69 DKK minimum is calculated as the ratio of weekly unemployment insurance benefits to average weekly hours (Arbejdsidektoratet, Copenhagen).
November 1994, and the number of these who stayed with the firm through to the following year. A summary of the sample statistics is shown in Table 1.

In constructing the firm wage rate and the person counts on which these statistics are based, only workers between the ages of 16 and 65 years of age are included. Because there are good reasons to believe that the hourly wages for some individuals were abnormally low and for others abnormally high due to measurement error, the firm average hourly wage was constructed after excluding the wages rates for certain individuals as follows: The wage of any worker for whom reported wage rates were less than 69 DKK per hour were excluded. This figure is regarded as an estimate of the effective legal minimum wage. The wage rate of any individual in the top one percent of the observed distribution was also excluded. Although these wage rates were excluded for the purpose of computing the firm wage average, the estimate of the firm’s wage policy, all workers were included in the employment and stayer number person counts.

The wage offer distribution, \( F \), and the wage earned distribution, \( G \), are constructed separately for each sub-sample. Specifically, for each firm, first an hourly wage paid is constructed by averaging the Statistics Denmark estimate of the hourly wage earned by each worker of the occupational type employed by the firm during the November 1994 to November 1995 year. Given this number, denoted \( w_i \) in the case of firm \( i \), \( F \) is constructed by weighting these by the fraction of all workers hired from non-employment (unemployment plus not in the labor force) by firm \( i \) during the year. The wage earned distribution, \( G \), uses the same firm wages but weights them by each firm’s relative employment size in November 1994.

In Table 1, the second row indicates that there are 113,325 firms employing at least one worker. However, there are only 49,667 firms employing at least 1 manager. The occupation “manager” excludes owner-operators because the definition of wage is problematic in such
Table 1: Sample Statistics

<table>
<thead>
<tr>
<th>Sample</th>
<th>Private</th>
<th>Managers</th>
<th>Salaried</th>
<th>Skilled</th>
<th>Unskilled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Size (# of firms)</td>
<td>113,325</td>
<td>49,667</td>
<td>57,513</td>
<td>44,527</td>
<td>70,886</td>
</tr>
<tr>
<td>Min Wage</td>
<td>69</td>
<td>69</td>
<td>69</td>
<td>69</td>
<td>69</td>
</tr>
<tr>
<td>Max Wage</td>
<td>435</td>
<td>626</td>
<td>323</td>
<td>310</td>
<td>331</td>
</tr>
<tr>
<td>Median Offer</td>
<td>132</td>
<td>188</td>
<td>124</td>
<td>138</td>
<td>115</td>
</tr>
<tr>
<td>Mean Wage Offer</td>
<td>138</td>
<td>188</td>
<td>128</td>
<td>141</td>
<td>121</td>
</tr>
<tr>
<td>Std. dev. of Wage Offer</td>
<td>32</td>
<td>50</td>
<td>25</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>Median Wage Earned</td>
<td>142</td>
<td>198</td>
<td>131</td>
<td>141</td>
<td>121</td>
</tr>
<tr>
<td>Mean Wage Earned</td>
<td>146</td>
<td>198</td>
<td>133</td>
<td>144</td>
<td>126</td>
</tr>
<tr>
<td>Std. dev. of Wage Earned</td>
<td>32</td>
<td>48</td>
<td>25</td>
<td>26</td>
<td>28</td>
</tr>
<tr>
<td>Min Size</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Max Size</td>
<td>15,870</td>
<td>4,069</td>
<td>7,163</td>
<td>1,708</td>
<td>8,856</td>
</tr>
<tr>
<td>Mean Size</td>
<td>13.36</td>
<td>6.20</td>
<td>6.22</td>
<td>5.94</td>
<td>7.81</td>
</tr>
<tr>
<td>Std. dev. of Size</td>
<td>125.84</td>
<td>45.19</td>
<td>70.25</td>
<td>28.09</td>
<td>64.50</td>
</tr>
<tr>
<td>Mean Stayers</td>
<td>9.26</td>
<td>4.83</td>
<td>4.59</td>
<td>4.31</td>
<td>4.78</td>
</tr>
<tr>
<td>Std. dev. of Stayers</td>
<td>96.90</td>
<td>39.43</td>
<td>58.04</td>
<td>23.01</td>
<td>41.26</td>
</tr>
</tbody>
</table>

cases. Denmark has a high fraction of small firms, which accounts for the difference between the two numbers.

4 Results

4.1 Private Sector

Before proceeding to the structural estimates, it is useful to examine what the raw data indicate about the relation between separations and wages. Figure 1 presents a non-parametric regression of the firm separation rate on $w$. The pointwise 95% confidence intervals are also displayed in the figure. As expected, the separation function is decreasing in the wage rate throughout its range. The decline is greatest in the lowest part of the wage distribution, namely those wages where $(1 - F(w))$ is large.

Turning to the structural analysis, parameter estimates of the separation function for the full sample of all private sector firms are reported in Table 2. The exogenous separation rate $\delta$ and the offer arrival parameter $\lambda$ are expressed as annual rates while the parameter $\gamma$ is the
elasticity of the search effort with respect to the expected economic payoff to search effort. Equivalently, its inverse $1/\gamma$ is the elasticity of the marginal cost function with respect to search effort. For reference, $\gamma = 1$ is the case of a quadratic search cost function.

The point estimates for the full sample are those obtained using the maximum likelihood procedure described above after substituting the constructed sample wage offer distribution for the market distribution, $F$. Although the reported standard errors are computed by taking the offer distribution $F$ as given, the results obtained by computing 95% confidence intervals for each parameter using bootstrap techniques confirm that the additional sampling variance attributable to the fact that $F$ is estimated by the empirical distribution function is negligible. All parameters are highly significant. Indeed, sample sizes are such that uncertainty only affects the precision of the estimates of the third significant digit.

The job destruction rate estimate, $\delta$, is 0.287 per year. This is roughly in accord with
aggregate U. S. experience. Bleakley et al. (1999) provide monthly separation rates for the U. S. that average 1.733% for 1968-1998, implying an annual turnover rate of 21%. According to the model, the estimate suggests that jobs last somewhat less than four years, abstracting from voluntary job to job movements that are sensitive to the employer’s relative wage. However, as an estimate of the flow of workers from employment to unemployment, it is almost three times higher than that obtained by Rosholm and Svarer (1999) using Danish worker panel data on transitions from employment to unemployment. Since in our estimation procedure we do not condition on the destination state of workers who leave the firm, one reasonable interpretation of the difference is that some workers move from one job to another without experiencing an intervening unemployment spell for reasons that have nothing to do with the relative wages in the two jobs. In short, \( \delta = \delta_0 + \delta_1 \) where \( \delta_0 \) represents transitions to unemployment while \( \delta_1 \) is the intercept in the job-to-job transition rate function.

Our estimate of \( \lambda \) for the full sample is 0.583 per year. As the sum, \( \delta + \lambda = 0.87 \), is the separation rate of workers employed in the lowest paying firm, the expected duration of a match paying the lowest wage in the market is only \( 1/0.87 = 1.149 \) years. However, as the wage earned increases the separation rate decreases, both because workers search less intensively and because higher paying jobs are more difficult to find.

The parameter \( \gamma \) in the economic model is the elasticity of search effort with respect to the expected return to search, which declines with the wage earned, and its inverse is the elasticity of the marginal cost function with respect to search effort. The estimate \( \gamma = 1.185 \) suggests a cost of effort function which is approximately quadratic. Note that the
data could have driven the estimate negative, in which case the economic interpretation of this parameter would be lost. Since $F(w)$ is increasing, equation (6) implies that $s(w)$ can increase even if the separation rate, $d(w)$, is decreasing. Indeed, in the received literature, search intensity while employed is assumed to be independent of the wage earned. This is equivalent to the prior specification $\gamma = 0$. This case is clearly rejected given the small standard error on our estimate of $\gamma$.

The steady state condition, equation (8), together with our estimates of the separation function parameter vector $(\delta, \gamma, \lambda)$ and the observed offer distribution $F$ can be used to compute an implied steady state distribution of wages earned, $G^*(w)$, which can be compared with the observed distribution, $G(w)$. However, the following question arises: Does the steady state relation continue to hold if $\delta$ is reinterpreted as the sum of the transition rate to non-employment and the intercept of the job-to-job transition rate? The answer is yes, provided that the wage earned on the new job by a worker who changes jobs for non-wage reasons can be regarded as a random draw from the offer distribution.

To prove the assertion, let $\delta = \delta_0 + \delta_1$ where $\delta_0$ is regarded as the rate of transition from employment to non-employment and $\delta_1$ is the intercept of the job-to-job transition rate function. Under the assumption that workers who move between jobs for non-wage reasons earn a random offer in the destination job, the flow of workers to jobs that pay $w$ or less is

$$s_0 \lambda F(w)u + \delta_1 F(w)(1 - u),$$

where the first term is the inflow from non-employment and the second term is the inflow from employment. Equating the inflow to the outflow yields an equation equivalent to (8)

$$\delta G(w) + \lambda[1 - F(w)] \int_w^w s(x)dG(x) = \frac{s_0 \lambda F(w)u + \delta_1 F(w)(1 - u)}{1 - u}$$

$$= (\delta_0 + \delta_1) F(w) = \delta F(w),$$
because the steady state non-employment rate solves

\[ \frac{s_0 \lambda u}{1 - u} = \delta_0. \]

The actual wage offer distribution, \( F(w) \), wage earned distribution, \( G(w) \), and estimated steady state distribution, \( G^*(w) \), are plotted in Figure 2. The wage offer distribution is represented by the curve that lies everywhere to the left of the other two, as the theory says it should. The estimated steady state wage distribution is represented by the curve containing the dots. It and the observed wage distribution, the remaining curve in the panel, are virtually coincident given the resolution of the chart. In short, for the structural parameters estimated, *the model explains the entire employment effect*, as represented by the magnitude of the horizontal difference between the distributions of wage earned and offered. The estimated employment effects are substantial. Indeed, from Table 1, the difference between the median wage earned and offered is 10 DKK per hour, about 7% of the 142 DKK median wage earned per hour. It should be pointed out that the well documented experience
and tenure effects on worker wages are not represented in the difference between our wage and offer distributions, at least not effects that represent accumulation of worker ability. As the firm wage rate used to construct the two distributions is an average of that paid to all workers, differences in tenure and experience characteristics across workers cancel to the extent that their distributions are the same across employers. Under this orthogonality condition, the horizontal difference between $F$ and $G$ is a general equilibrium effect that exists if and only if wages are dispersed and workers flow from lower to higher paying firms. If the orthogonality condition fails, we mistakenly include tenure effects in the employment effect. But as observed in section 1, outside estimates of the tenure effect are small, and we may still attribute the bulk of the observed employment effect to mobility.

It is useful to note that the closeness of the implied distribution of earnings to the actual distribution ($G^*$ and $G$) tells us how accurate the steady state assumption is. Related empirical work has generally imposed steady state conditions because data limitations precluded observation of wage offers. That is, although the estimation methods used in search models do not require the assumption of ergodicity, using the estimates to compute distributions of, say, employment durations or earnings typically do require this assumption. Testing this assumption is difficult. Here, we provide a natural test by comparing the actual distribution of earnings with the steady-state distribution implied by the parameter estimates.

### 4.2 Stratification by Occupation

Estimating the model by pooling all the workers employed in the private sector obviously ignores the possible importance of worker heterogeneity. There are at least two reasons why differences in worker types should be taken into account: First, the structural parameters of interest may simply vary by type. Second, the worker composition by type may differ across firms. The second reason for stratifying the sample by type may actually be more important than the inappropriate aggregation implied by the first reason because ignoring it can induce
sources of measurement error that bias the parameter estimates even if the true values were equal across types.

To illustrate the possible source of aggregation error of the second kind, consider the following specification of the wage. Let the index $i$ represent a firm in the sample and $j$ a worker type and assume that the wage paid by firm $i$ can be decomposed into a fixed firm and a fixed type effect as follows:

$$w_{ij} = \mu_j + \epsilon_i$$

In other words, $\mu_j$ is the common component of the wage paid by all firms to workers of type $j$ and $\epsilon_i$ is the firm’s wage differential. Obviously, because the average wage paid by firm $i$ is

$$\bar{w}_i = \sum_j \theta_{ij} \mu_j + \epsilon_i,$$

where $\theta_{ij}$ represents the share of firm $i$’s employees who are of type $j$, differences in the measured firm wage reflect true differentials if and only if the worker type composition is the same across firms. When this condition fails, an employer who disproportionately employs higher wage types will be inappropriately regarded as a high wage employer even if actual differentials in $\epsilon_i$ are distributed independently of the worker type composition across the firms. Since in this case observed differentials exceed actual, the measured wage offer distribution is more dispersed that the actual. Given the non-linear relationships in the model, the exact direction of the bias induced by this form of measurement error is not obvious. Still, its existence suggests that correcting for worker heterogeneity may be important.

The estimation results and their implications are reported for each of four occupation sub-samples. The four occupational categories, managers, salaried workers, skilled and unskilled workers, are mutually exclusive and exhaustive, as already noted. Although the information available on worker characteristics found in the IDA data is much richer, an initial stratification by occupation provides a fair test of whether aggregation bias of the type suggested
above is important. First, one would expect occupational composition to differ across employers for a variety of reasons. In addition, the magnitude of the wage differential offered by a given firm is also likely to depend on the occupation. Finally, potentially important occupational differences in job destruction rates as well as occupational variation in demand conditions and search costs can also be anticipated.

The structural parameter estimates (with estimated asymptotic standard errors in parentheses) are reported in Table 3 for the occupation sub-samples. For comparison, the parameter estimates derived from the full private sector sample are included in the first column.

Estimates of the exogenous separation rate parameter $\delta$ fall with the level of the occupation as ranked by the skill-education hierarchy. This observation seems to be consistent with the general fact that layoffs are higher for the less skilled and less educated. Of course, there is no particular reason to see the same relationship for job-to-job transitions not related to employer wage differentials, the other possible component included in the estimated parameter. The fact that the estimate for the full sample lies between the two highest and the two lowest sub-sample estimates suggests that the possible aggregation bias due to cross employer composition effects discussed above are not particularly important for obtaining an estimate of this parameter with the full sample. However, the negligible sampling error suggested by the standard error estimates indicates that the differences across sub-samples in the estimates are nonetheless real.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Private $\delta$</th>
<th>Managers $\delta$</th>
<th>Salaried $\delta$</th>
<th>Skilled $\delta$</th>
<th>Unskilled $\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.2873</td>
<td>0.2162</td>
<td>0.2392</td>
<td>0.3007</td>
<td>0.3950</td>
</tr>
<tr>
<td></td>
<td>(0.0007)</td>
<td>(0.0013)</td>
<td>(0.0014)</td>
<td>(0.0016)</td>
<td>(0.0018)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.1855</td>
<td>1.4919</td>
<td>1.0789</td>
<td>2.4390</td>
<td>0.7686</td>
</tr>
<tr>
<td></td>
<td>(0.0198)</td>
<td>(0.0605)</td>
<td>(0.0365)</td>
<td>(0.1281)</td>
<td>(0.0319)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.5833</td>
<td>0.3211</td>
<td>0.4418</td>
<td>0.4585</td>
<td>0.4787</td>
</tr>
<tr>
<td></td>
<td>(0.0055)</td>
<td>(0.0090)</td>
<td>(0.0089)</td>
<td>(0.0218)</td>
<td>(0.0080)</td>
</tr>
</tbody>
</table>
The estimates of the offer arrival rate parameter decrease with skill and education requirements. This result is consistent with the fact that more educated and skilled workers typically experience shorter unemployment spells. However, note that the full sample estimate of $\lambda$ is substantially larger than all of the sub-sample estimates. It is possible that this fact is a consequence of composition bias in the pooled sample. If so, the estimates for the sub-samples may also be biased upward to the extent that accounting for occupation does not fully correct for worker heterogeneity.

Although the cross sample estimates of $\gamma$, the elasticity of search effort with respect to its expected return, vary considerably over the occupations, the variation is not systematically associated with differences in the skill and education requirements for occupational membership. The full sample estimate of 1.185 is similar to those of both managers and salaried workers, while search effort is more responsive to expected return in the case of skilled workers and less responsive in the case of unskilled workers. Given the parametric specification, these differences are attributed to cross occupation differences in the curvature of the marginal cost of search effort function. The implication is that the marginal cost of search rises more steeply with effort in the case of unskilled workers than in any of the other occupations, and rises least for skilled workers. Considering the Danish labor market, these findings may reflect the fact that skilled workers participate in much better connected occupational networks than unskilled workers. In sum, then, search effort seems to be quite elastic with respect to its expected return in all the occupations, is highly sensitive in the case of skilled workers, and is somewhat less responsive than average in the case of unskilled workers.

Stratification by occupation makes sense to the extent that workers do not change occupation very easily. In fact, they do not in these data. For managers, 86% of all job movers retained their occupation. For salaried workers, 79% of movers stayed salaried, while for
Table 4: Employment Effects; Actual and Percent Explained  
(Wages in Danish Crowns per Hour)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Private</th>
<th>Managers</th>
<th>Salaried</th>
<th>Skilled</th>
<th>Unskilled</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st quartile of $F$</td>
<td>115.90</td>
<td>158.10</td>
<td>111.71</td>
<td>124.33</td>
<td>103.59</td>
</tr>
<tr>
<td>1st quartile of $G$</td>
<td>123.95</td>
<td>169.99</td>
<td>115.79</td>
<td>126.71</td>
<td>108.10</td>
</tr>
<tr>
<td>1st quartile of $G^*$</td>
<td>125.00</td>
<td>167.55</td>
<td>118.88</td>
<td>126.79</td>
<td>108.93</td>
</tr>
<tr>
<td><strong>1st quartile effect</strong></td>
<td><strong>8.05</strong></td>
<td><strong>11.90</strong></td>
<td><strong>4.08</strong></td>
<td><strong>2.38</strong></td>
<td><strong>4.51</strong></td>
</tr>
<tr>
<td>% explained</td>
<td>113%</td>
<td>79%</td>
<td>176%</td>
<td>103%</td>
<td>118%</td>
</tr>
<tr>
<td>2d quartile of $F$</td>
<td>132.00</td>
<td>187.86</td>
<td>124.18</td>
<td>137.85</td>
<td>115.05</td>
</tr>
<tr>
<td>2d quartile of $G$</td>
<td>142.18</td>
<td>198.04</td>
<td>131.29</td>
<td>141.47</td>
<td>121.11</td>
</tr>
<tr>
<td>2d quartile of $G^*$</td>
<td>141.67</td>
<td>196.38</td>
<td>131.20</td>
<td>140.64</td>
<td>121.41</td>
</tr>
<tr>
<td><strong>2d quartile effect</strong></td>
<td><strong>10.18</strong></td>
<td><strong>10.18</strong></td>
<td><strong>7.11</strong></td>
<td><strong>3.62</strong></td>
<td><strong>6.07</strong></td>
</tr>
<tr>
<td>% explained</td>
<td>95%</td>
<td>84%</td>
<td>99%</td>
<td>77%</td>
<td>105%</td>
</tr>
<tr>
<td>3d quartile of $F$</td>
<td>153.70</td>
<td>217.03</td>
<td>140.04</td>
<td>154.58</td>
<td>132.71</td>
</tr>
<tr>
<td>3d quartile of $G$</td>
<td>162.74</td>
<td>224.92</td>
<td>144.35</td>
<td>157.35</td>
<td>140.01</td>
</tr>
<tr>
<td>3d quartile of $G^*$</td>
<td>163.70</td>
<td>223.64</td>
<td>146.80</td>
<td>156.30</td>
<td>139.67</td>
</tr>
<tr>
<td><strong>3d quartile effect</strong></td>
<td><strong>9.04</strong></td>
<td><strong>7.89</strong></td>
<td><strong>4.30</strong></td>
<td><strong>2.78</strong></td>
<td><strong>7.30</strong></td>
</tr>
<tr>
<td>% explained</td>
<td>111%</td>
<td>84%</td>
<td>157%</td>
<td>62%</td>
<td>95%</td>
</tr>
</tbody>
</table>

skilled and unskilled workers the comparable rates were 84% and 82%. We also re-estimated the model excluding all occupation changers from the data, but this had little effect on the estimates.

Although the structural parameters generally differ across occupational sub-samples, the estimated model explains almost all of the employment effect measured at the median wage in all four cases. The graphical evidence for this assertion is illustrated in Figure 3. In the figure, the offer cdf $F(w)$ is at the far left in all cases, the wage distribution $G(w)$ is the curve on the right represented by a solid line and the steady state wage cdf $G^*(w)$ implied by the estimates and the offer distribution is represented by the curve with dots.

Table 4 provides a more quantitative comparison of how well the model explains the employment effect at each of the three quartiles for each of the individual occupations. The results for the pooled estimates are also reported for comparison. In the table, the employment effect is defined as the difference between the wage paid and the wage offered at each quartile. In each case, the percent explained is the ratio of the employment effect
predicted by the estimates and the actual employment effect as defined above.

For the private sector as a whole, the predicted median wage paid is almost identical to the actual, but the model over predicts the difference between wages paid and offered at both the 1st and the 3rd quartile. Across the occupational sub-samples, the model predicts the median wage paid to salaried and to unskilled workers, but under predicts the median wage paid to both managers and skilled workers. Although the model’s under prediction holds at all quartiles for managers, the model over predicts at both the 1st and 3rd quartiles in the case of salaried workers, where the prediction is exact at the median. In the case of unskilled
workers, the employment effect is explained at all quartiles. In the case of skilled workers, both the median and the spread are under predicted by the model. Put differently, the model works least well for skilled workers. In the Danish context “skilled” workers should be read as “unionized” workers. To the extent that their wages are set by collective bargaining, there are reasons to believe that the model would not work well. Indeed, U.S. evidence suggests that unions attempt to reduce inter-firm wage differentials. This could be one reason why the employment effect, as well as the explained portion thereof, is small for skilled workers.

4.2.1 Alternative Parameter Estimates

In this section we reestimate the model using the observations on the distribution of wages earned rather than wages offered. This is done by using the fact that the model and the wage distribution imply that the offer distribution is

$$F(w) = \frac{\delta G(w) + \lambda \int_w^w s(x)dG(x)}{\delta + \lambda \int_w^w s(x)dG(x)}.$$  \hspace{1cm} (16)

The parameter estimates obtained by imposing this condition on $F$ in equations (10) and (15) and using the observed distribution of wages earned, $G$, instead of the wage offer distribution, $F$, are reported in Table 5.

These alternative estimates provide an additional way of determining whether the steady state condition is violated in any material sense. As it turns out, the alternative estimates tell exactly the same economic story as the original estimates in Table 3. Specifically, the

### Table 5: Alternative Parameter Point Estimates (Std Errors)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Private</th>
<th>Managers</th>
<th>Salaried</th>
<th>Skilled</th>
<th>Unskilled</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>0.2872</td>
<td>0.2162</td>
<td>0.2395</td>
<td>0.3004</td>
<td>0.3932</td>
</tr>
<tr>
<td></td>
<td>(0.0010)</td>
<td>(0.0012)</td>
<td>(0.0015)</td>
<td>(0.0016)</td>
<td>(0.0018)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.1225</td>
<td>1.5089</td>
<td>0.9587</td>
<td>2.4745</td>
<td>0.6986</td>
</tr>
<tr>
<td></td>
<td>(0.0217)</td>
<td>(0.0633)</td>
<td>(0.0417)</td>
<td>(0.1243)</td>
<td>(0.0343)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.5899</td>
<td>0.3279</td>
<td>0.4482</td>
<td>0.4517</td>
<td>0.4892</td>
</tr>
<tr>
<td></td>
<td>(0.0056)</td>
<td>(0.0096)</td>
<td>(0.0096)</td>
<td>(0.0197)</td>
<td>(0.0083)</td>
</tr>
</tbody>
</table>
exogenous separation rate $\delta$ and the offer arrival rate at the smallest wage $\lambda$ both decline with occupational status. The point estimates of the search cost curvature parameter are essentially the same values, and are ranked across occupations in exactly the same order as were the original estimates. We conclude, therefore, that there is strong evidence for the steady state relationship implied by the model in our data set.

### 4.3 Robustness Checks

We have used the employment effect, defined above as the horizontal distance between the observed earnings distribution and the implied steady-state distribution, as the metric for judging how well the model explains wages. Of course the model could be correct but this might not be seen in this metric if the observed distribution of earnings was not yet close to the steady-state distribution. To investigate how well the model does fit the wage data we perform two further experiments. First, we calculate the first four central moments of the implied distribution of earnings and compare them to their sample equivalents. Second, we reestimate the model assuming, as is conventional in most search models, that the search intensity of workers is a constant, not influenced by their current wage. The results of these two experiments are shown in Table 6.

The table presents the comparisons for all private sector workers and the four occupational groups. The first line (Wage Earned) contains the sample moments of observed earnings; the second line contains the calculated moments from the implied steady-state distribution, $G^*$, and the third line, $G^{**}$, contains the moments of the steady state distribution implied by the model that constrains search intensity to be identical across workers. The positive values of skewness indicate that in Denmark as in most countries the wage distribution has

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11Table 6 reports the third and fourth central moments in their standardized form: i.e. $\alpha_3 = \mu_3 / (\mu_2)^{3/2}$ and $\alpha_4 = \mu_4 / \mu_2^2$, where $\mu_r$ is the $r$-th central moment. A probability distribution is positively skewed, negatively skewed, or symmetric as $\alpha_3 >, <$ or $= 0$. A distribution has heavy tails, thin tails or normal tails as $\alpha_4 >, <$, or $= 3$.
Table 6: Moments of the Earnings Distribution.

<table>
<thead>
<tr>
<th>Group</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Private</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage Earned</td>
<td>146</td>
<td>32</td>
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<tr>
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<td>8.8</td>
</tr>
<tr>
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<td>153</td>
<td>37</td>
<td>1.4</td>
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<tr>
<td><strong>Managers</strong></td>
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<td>47</td>
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<td>8.4</td>
</tr>
<tr>
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<td>48</td>
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<td>9.5</td>
</tr>
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<td>204</td>
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<td>1.1</td>
<td>6.8</td>
</tr>
<tr>
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<tr>
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<tr>
<td><strong>Skilled</strong></td>
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</tr>
<tr>
<td>$G^*$</td>
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<td>25</td>
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<td>5.2</td>
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</tr>
<tr>
<td>$G^*$</td>
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<td>28</td>
<td>1.8</td>
<td>9.5</td>
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<td>30</td>
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</table>

a long tail to the right. The kurtosis values of 5-10 inform us that the wage distribution is heavy-tailed relative to the normal distribution’s value of 3. Clearly the third and fourth moments are fit reasonably well by either $G^*$ or $G^{**}$, with the latter generally doing slightly better. However, the first two moments are fit substantially better by the variable search intensity model, $G^*$. The results are consistent with the notions that the labor market is in steady state, and on-the-job search effort is a declining function of the current wage.

## 5 Conclusions

Establishing a structural link between two well known empirical observations, that higher paying employers have lower turnover and that workers with more experience earn higher wages, is a principal empirical contribution of the paper. Given the existence of wage policy dispersion across employers, a link is implied by the fact that workers have an incentive to
seek higher paying jobs. If they do so, workers flow from low to high paying firms and, consequently, the wage earned is positively related to the time since the last unemployment spell.

The empirical exercise conducted in the paper is one of estimating the parameters of a specific structural model of turnover using firm level observations on separation flows and wages, and the distribution of alternative wage offers. The model is the standard on-the-job search formulation with endogenous search effort. The exercise is successful in that it yields well determined coefficient estimates that are consistent with the theory for both the full sample and for each of the four occupational sub-samples.

The estimates strongly support the hypothesis that workers choose search effort in response to economic incentives. Specifically, the high estimated elasticities of search effort with respect to expected return to search ($\gamma$) imply that a worker searches more when earning a relatively low wage because the return is higher. These results suggest that one should incorporate this feature in future empirical work on worker turnover.

When workers flow from lower to higher paying jobs without intervening spells of non-employment, the expected wage earned rises with experience as measured by the elapsed time since the last non-employment spell. The impact of this measure of experience on the wages of individual workers is reflected at the market level by the employment effect, defined as the horizontal difference between the distribution of wages earned by the employed and the distribution of wages offered applicants. Conditional on the wage offer distribution and the structural parameter values, the model can be used to predict the employment effect. Since the wage distribution itself was not used in the estimation of the model’s parameters, these predictions provide an out of sample test of the theory.

For the full sample of all workplaces with workers not distinguished by occupation, the theory passes the test with flying colors. Indeed, the predicted difference between the median

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wage earned and offered is essentially identical to the actual difference. Of course, there are
differences in the extent to which the model explains the employment effect, both across
occupations and across the quartiles used to measure the effect within occupations. The
model explains all of the difference between the median wage and offer for salaried and
unskilled workers and about 80% to 85% of the difference for managers and skilled workers.
Finally, the model under predicts the employment effect by about 15% at all quartiles in the
case of managers.

These findings provide ample evidence that labor market imperfections have an important
influence on the distribution of wage income. Separations, however, are but one part of the
story. Reducing turnover lessens the need to use wages as a recruitment tool, but does not
eliminate it. Indeed, firm wage policy has to balance investment decisions by workers and
firms in firm-specific capital with turnover considerations. Linking the hiring and separation
problems faced by workers and firms remains a challenging problem.
References


Bowlus, Audra J., Nicholas M. Kiefer, and George R. Neumann (2001). Equilibrium search models and the transition from school to work. *International Economic Re-


